μ JADE ε : Micro adaptive differential evolution to solve constrained optimization problems

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Abstract—A highly competitive micro evolutionary algorithm to solve unconstrained optimization problems called μ JADE (micro adaptive differential evolution), is adapted to deal with constrained search spaces. Two constraint-handling techniques (the feasibility rules and the ε -constrained method) are tested in μ JADE and their performance is analyzed. The most competitive version is then compared against two highly-competitive algorithms for constrained optimization when solving a wellknown set of 36 test problems, and also against a small population algorithm tested on another well-known set of thirteen problems. The results show that μ JADE provides a better performance when coupled with the ε -constrained method and also that its results are competitive against those provided by state-of-the-art approaches.

I. INTRODUCTION

Several real-world optimization problems are hard to solve [1]. Normally, those problems have constraints that delimit the feasible region within the whole search space, and the search algorithm must find it, sample it, and find the best feasible solution. Evolutionary algorithms (EAs) have been very competitive to deal with such search problems, and differential evolution (DE) has been particularly successful [2].

There are research efforts on the usage of DE with small populations (*micro differential evolution*, μDE), but mainly focused on unconstrained optimization problems. Caraffini et al. in [3] applied a μDE to solve problems with high dimensionality using a simple local search, which consisted in moving the best vector along the axes. Such mechanism was applied at every generation. They called this algorithm μ DEA. In their paper, the algorithm was compared against stateof-the-art DE variants like Self-Adaptive Differential Evolution (SADE), Adaptive Differential Evolution (JADE) and Modified Differential Evolution with p-Best Crossover (MDEpBX), in four different benchmarks (CEC2005, BBOB2010, CEC2008, CEC2010). The result of the comparison indicated that μ DEA outperformed all the algorithms aforementioned, concluding that small populations can work well with DE in unconstrained search spaces.

Another μ DE was proposed by Salehinejad et al. in [4], where they changed the concept of the mutation factor by converting it from a real number into a vector, i.e., a different random value for the mutation factor was generated for each mutation applied to the population. The authors called their proposal *Micro Differential Evolution with Vectorized random Mutation factor (MDEVM)*. MDEVM was compared against the Standard Micro-Differential Evolution (SMDE) and the Micro-differential evolution with scalar random mutation factor (MDESM) in 28 functions of the CEC 2013 benchmark. The results showed that MDEVM had a higher performance than those of the compared algorithms. Such conclusion remarks the viability of using a small population in DE to solve high-dimensional problems.

In [5], Brown et al. proposed a small population version of the Self-adaptive differential evolution (μ JADE) for unconstrained continuous optimization. μ JADE uses an adaptive mechanism of the F and CR parameters, similar to the original JADE, i.e., adapting their values every max(100, 10D)generations and proposing a new mutation operator denoted as current-by-rand-to-pbest/1. This approach was tested in the classical multimodal benchmark in 30 dimensions and it was compared against some variants that use conventional sized populations [6]. The results showed that μ JADE is more reliable than state-of-the-art DE algorithms.

It is important to mention that, the main limitation of the μ EAs is the difficulty maintaining the diversity of solutions. This limitation causes a premature convergence on possible local optima due to the small populations. This problem hardly occurs in conventionally population sized algorithms. Most of the μ EAs have implemented mechanisms to deal with this situation, getting competitive results in the context where they were tested.

On the other hand, there are also works on μ EAs for constrained spaces. In [7], Fuentes-Cabrera et al. implemented a local version of particle swarm optimization with a small population (μ PSO) to solve the thirteen functions described in [8].

As it can be seen, μ DE has been, to the best of the authors' knowledge, extensively tested on unconstrained search spaces, but overlooked or scarcely analyzed in constrained search spaces. For that reason, the main motivation of this work lies on studying the behavior of one of the μ DE algorithms, μ JADE in this case, when solving numerical constrained problems. Two constraint-handling mechanisms are added to μ JADE and their performance is studied.

The rest of this document is divided as follows: Section II introduces the basics on μ JADE. Section III describes the constraint-handling techniques added in μ JADE. Section IV presents the experiments and results obtained. Finally, Section V includes the conclusions and future work.

II. μ JADE: MICRO ADAPTIVE DIFFERENTIAL EVOLUTION

 μ JADE has four elements which are described in the next subsections.

A. Mutation operator

The mutation operator introduced in μ JADE is shown in Equation 1 and it is denoted as current-by-rand-to-pbest/1.

$$v_i = x_i + F_i(x_{best}^p - x_a) + F_i(x_b - \tilde{x_c})$$
(1)

where v_i is the mutation vector to be generated, x_i is the target vector, F_i is the current value for the mutation factor (detailed later), \boldsymbol{x}_{best}^p is a randomly chosen vector from the the best (3/NP)% vectors of the population, x_a and x_b are vectors chosen randomly from the current population, and $\tilde{x_c}$ is a vector randomly chosen from the union of the current population and the auxiliary file where vectors are stored. The main idea of this operator was to improve the exploratory capacity of small populations maintaining a good convergence. When $x_i \not\approx x_a$ and $x_i \neq x_a$, current-by-rand-to-pbest/1 is exploratory. However, when $x_i \approx x_a$, current-by-randtopbest/1 is similar to current-to-pbest/1. The objective was to accelerate convergence at the end of the optimization process and reducing the probability of finding local optimum in early stages of optimization. Moreover, two constraints used in the original JADE were removed: (1) $\tilde{x_c} \neq x_b$, and (2) $\tilde{x_c} \neq x_a$, both to avoid a greedy behavior of the operator.

B. F and CR adaptation

 μ JADE introduced a modification in the adaptation of the F and CR parameters to prevent a quick value decrease at early generations and avoiding the diversity loss on small population algorithms. This modification consists in the updating process of F and CR at every max(100, 10D) generations rather than at each generation, where D is the dimensionality of the optimization problem.

C. Perturbation

A perturbation was introduced to provide μ JADE with the ability to escape from local optima and increase diversity. This perturbation was proposed by Farfaj et al. in [9], and it is incorporated after the crossover operator as in Equations 2 and 3. To indicate that the perturbation was applied to a particular variable of a given vector, it is marked with a 0 in a binary vector *b* as shown in Equation 4.

$$r_{i,j} = rand(0,1) \tag{2}$$

$$u_{i,j} = \begin{cases} L_j + rand(0,1)(U_j - L_j) & \text{if} \quad r_{i,j} \leq 0.005\\ u_{i,j} & otherwise \end{cases}$$
(3)

$$b_{i,j} = \begin{cases} 0 & \text{if} \quad r_{i,j} \le 0.005\\ b_{i,j} & otherwise \end{cases}$$
(4)

D. Restarts

One way to avoid stagnation on μ EAs is restarting the population at every generation. Despite the aforementioned modifications, it is possible that the search stagnates on local optima. For that reason, the last modification for μ JADE was to apply this mechanism at every max(1000, 100D) generations if the best solution did not improve. It is worth noticing that the restart mechanism is applied by excluding the best vector in the current population.

The complete pseudocode of μ JADE is shown in Algorithm 1.

III. CONSTRAINT-HANDLING TECHNIQUES

Different constraint-handling techniques are reported in the specialized literature of bio-inspired algorithms [2]. In this section, two competitive techniques are introduced and adopted in this research: the *feasibility rules* and the ε -constrained method. Those techniques were selected because their competitive performance and also because they modify only the selection criteria in μ JADE, i.e., they do not significantly change the algorithm [2].

A. Feasibility Rules

The set of three feasibility rules is one of the most popular constraint-handlers [10]. This mechanism consists in three selection criteria as follows: (1) when two feasible individuals are compared, the individual with the best objective function value is chosen, (2) if one of the individuals is infeasible and the other one is feasible, then the feasible individual is chosen, and (3) when two infeasible individuals are compared, the individual with the lowest sum of constraint violation is chosen. The idea is to favor good feasibles individuals .

B. ε -constrained method

Takahama et al. in [11] introduced the ε -constrained method, which converts a constrained optimization problem into an unconstrained optimization problem. To evaluate if an individual is feasible or not, the ε -constrained method defines a *constraint violation* $\phi(x)$ whose value is the same as the sum of constraint violation used in the feasibility rules, see Equation 5:

$$\phi(x) = \sum_{j} \|max\{0, g_j(x)\}\|^p + \sum_{j} \|h_j(x)\|^p$$
(5)

where p is a positive number, 2 in this case. Furthermore, ε level comparisons are defined as an order relation on a pair of objective function constraint violation values $(f(x), \phi(x))$. Let $f_1(f_2)$ and $\phi_1(\phi_2)$ be the objective function and constraint violation values of vectors $x_1(x_2)$, respectively. The ε level comparisons are defined in Equation 6:

$$(f_1,\phi_1) \leqslant_{\varepsilon} (f_2,\phi_2) \Leftrightarrow \begin{cases} f_1 \leqslant f_2 & \text{if} \quad \phi_1,\phi_2 \leqslant \varepsilon \\ f_1 \leqslant f_2 & \text{if} \quad \phi_1 = \phi_2 \\ \phi_1 < \phi_2 & otherwise \end{cases}$$
(6)

It is worth noticing that when $\varepsilon = \infty$ two infeasible vectors are compared based on their objective function values. In

Algorithm 1 μ JADE

1: Initialise population (P) 2: $\mu_{CR} = 0.5$ 3: $\mu_F = 0.5$ 4: Initialize file (A) empty s: for g = 1 to number of generation do for i = 1 to NP do 6: $CR_i = randn_i(\mu_{CR}, 0.1)$ 7: $F_i = randc_i(\mu_F, 0.1)$ 8: Randomly select $x_a \neq x_i$ 9 Randomly select $x_b \neq x_a \neq x_i$ 10: Randomly select $x_{best}^p \neq x_a$ from pNP best popu-11: lation vectors Randomly select x_c from $P \cup A$ 12: Randomly select $j_{rand} \in \mathbb{N}_{\leq D}^+$ 13: $\begin{aligned} v_i &= x_i + F_i(x_{best}^p - x_a) + F_i(x_b - \tilde{x_c}) \\ v_{i,j} &= \begin{cases} \frac{L_j + x_{i,j}}{2} & \text{if } v_{i,j} < L_j \\ \frac{U_j + x_{i,j}}{2} & \text{if } v_{i,j} > U_j \\ v_{i,j} & otherwise \end{cases} \end{aligned}$ 14: 15: for j = 1 to D do 16: $d_{i,i} = rand(0,1)$ 17: $\begin{aligned} u_{i,j} &= \begin{cases} v_{i,j} & \text{if } OR(d_{i,j} < CR_i, j = j_{rand}) \\ x_{i,j} & \text{otherwise} \end{cases} \\ b_{i,j} &= \begin{cases} 1 & \text{if } OR(d_{i,j} < CR_i, j = j_{rand}) \\ 1 & \text{if } OR(d_{i,j} < CR_i, j = j_{rand}) \end{cases} \end{aligned}$ 18: $b_{i,j} =$ 19: ĴΟ. end for 20: for j = 1 to D do 21: $r_{i,i} = rand(0,1)$ 22: $u_{i,j} = \begin{cases} L_j + rand(0,1)(U_j - L_j) \\ u_{i,j} \end{cases}$ if $r_{i,j} \leq 0.005$ 23: $\begin{array}{c} u_{i,j} \\ \text{if} \quad r_{i,j} \leqslant 0.005 \\ otherwise \end{array}$ otherwise $b_{i,j} = \begin{cases} 0\\ b_{i,j} \end{cases}$ 24 end for 25 end for $CR_i = \frac{\sum_{j=1}^{D} b_{i,j}}{D}$ 26 if $f(u_i) < f(x_i)$ then 27: $x_i \to A$ 28 $x_i = u_i$ 29: $CR_i \rightarrow S_{CR}$ 30 $F_i \rightarrow S_F$ 31: if u_i is better than best population vector then 32 BIR = BIR + 133: end if 34 end if 35: end for 36: Randomly remove vectors from A so that $|A| \leq NP$ 37: if mod(g, max(100, 10D)) = 0 then 38: $\mu_{CR} = (1 - c)\mu_{CR} + cL_1(S_{CR}), L_1(\emptyset) = 0$ 39: $\mu_F = (1 - c)\mu_F + cL_2(S_F), L_2(\emptyset) = 0$ 40: $S_{CR} = S_F = \emptyset$ 41: end if 42: if mod(g, max(1000, 100D)) = 0 then 43: if BIR= 0 then 44: Reinitialize population excluding the best vector. 45: end if 46: end if 47: 48: end for

contrast, when $\varepsilon = 0$, the two infeasible vectors are compared based on their constraint violations.

The ε level is controlled using Equation 7, where $\varepsilon(0)$ is initialized with the constraint violation of the θ -th vector in the population (x_{θ}) . ε is updated as long as the number of iterations t does not exceed the value T_c , after that, ε is set to 0.

$$\varepsilon(0) = \phi(x_{\theta})$$

$$\varepsilon(t) = \begin{cases} \varepsilon(0)(1 - \frac{t}{T_c})^{cp} & , \quad 0 < t < T_c, \\ 0 & t \ge Tc \end{cases}$$
(7)

C. Adding the constraint-handling techniques to μ JADE

The addition of each one of the two constraint-handling techniques into μ JADE is straightforward because both of them are just comparison criteria which replace the original μ JADE criterion based only in the objective function value for unconstrained optimization, e.g., lines 27, 32, and 45 in Algorithm 1.

IV. EXPERIMENTS AND RESULTS

Three experiments were carried out to evaluate the performance of μ JADE with each constraint-handling technique. For the first two experiments, a well-known set of constrained optimization problems with 18 test problems with 10 and 30 dimensions was used [12]; meanwhile for the third experiment, a well-known set of thirteen constrained problems was used [13]. In the first experiment, the two constraint-handling techniques aforementioned were compared to identify that with the best performance: μ JADE (FR) is μ JADE with the feasibility rules and μ JADE ε is μ JADE with the ε -constrained method. In the second experiment, the most competitive algorithm of the first experiment was compared against two highly competitive EAs for constrained optimization. Those algorithms are the ε DEag proposed by Takahama et al. in [11], and DEwAPI proposed by Elsayed et al. in [14]. Finally, in the third experiment, μ JADE ε was compared against μ PSO to validate the performance in the world of small population size algorithms.

The 95%-confidence Wilcoxon Signed Rank Test was applied to the samples of results, where three symbols were used: "=", "+" and "-"; where "=" indicates that there is no significant difference between the two algorithms, "+" denotes that the first algorithm is significantly better than the second one, and "-" means that the first algorithm is significantly worse than the second algorithm.

A. First experiment: μ JADE (FR) vs. μ JADE ε

The parameters used by μ JADE using both constrainthandling techniques are summarized in Table I, in order to keep the same execution environment. The number of evaluations was 200000 for 10D test problems and 600000 for 30D test problems. Those parameters were obtained by preliminary experiments.

TABLE I PARAMETER SETTING FOR μ JADE (FR) and μ JADE ϵ .

Parameter	Value
Population size	8
с	1.5
T_c	4000
ср	5

Tables II and III show the results found by each μ JADE version for 10D and 30D. In both dimensionalities, μ JADE ε obtained better results than μ JADE (FR) based on the Wilcoxon test. For 10D, μ JADE ε was significantly better in 11 test problems out of 18, in the remaining ones, there were no significant differences. In the same way, for 30D, μ JADE ε was significantly better in 10 test problems out of 18, while in the other seven, no significant differences were observed. C04 in 30D was omitted because both versions were unable to find feasible solutions.

Four measures were used to increase the empirical comparison to determine which constraint-handling technique performed better: Feasibility Probability (FP), Probability of convergence (P), Average number of Function Evaluations (AFES) and Successful Performance (SP) [15]. FP represents the number of feasible executions (i.e., those runs with at least one feasible solution found) divided by the total number of independent executions; FP goes from 0 to 1, where 1 indicates that each independent run reached the feasible region. P is calculated with the number of successful runs (i.e. those runs where the neighborhood of the best feasible solution was reached) divided by the total number of independent executions. To determine the closeness to the best feasible solution, a small tolerance, i.e., 1E-4, was used. P goes from 0 to 1, where 1 denotes that all independent executions were successful runs. AFES is calculated by averaging the number of evaluations required by a successful run to reach the best known feasible value. For AFES, lower values are preferred. Finally, SP is a combination of AFES and P in order to measure the speed and reliability of the algorithm It is calculated by dividing AFES from P. As a result, lower values are preferred because they mean a good compromise between speed and consistency of the algorithm. It is important to mention that the best values used to calculate the successful runs in P, AFES, and SP were obtained from [11]. The results for those four measures are included in Tables II and III for 10D and 30D, respectively.

The results of those measures indicate that μ JADE ε outperformed μ JADE (FR), because the first one presented better results in most test problems. For FP, both algorithms had similar results. However, only in two test problems μ JADE (FR) reached higher values with respect to those of μ JADE ε , having a better chance of finding feasible solutions. Regarding other measures μ JADE ε provided superior results, being more capable to find a successful solution in less time, i.e., μ JADE ε provided better speed and reliability compared with μ JADE (FR).



Fig. 1. Convergence plots of $\mu JADE$ (FR) and $\mu JADE\varepsilon$ for 10D C02 test problem.



Fig. 2. Convergence plots of μ JADE (FR) and μ JADE ϵ for 30D C01 test problem.

Finally, Figs. 1 and 2 include two representative convergence plots where μ JADE ε (red line) provided a faster approach to a good solution. Such behavior is easier to see in 10D than in 30D. For all the above mentioned results, μ JADE ε was chosen as the most competitive approach, which will be compared in the next experiment against highly competitive algorithms for constrained optimization.

B. Second experiment: $\mu JADE \varepsilon$ vs. conventional sized population algorithms

 μ JADE ε was compared against two competitive conventional sized population algorithms: ε DEag proposed by Takahama et al. in [11] and *DEwAPI* proposed by Elsayed et al.

T	ABLE II
10D results by μ JADE (FR) and μ JADE ϵ . B	BOLD NUMBERS REPRESENT THE BEST VALUES FOUND.

Function	Algorithm	Best	Worst	Median	Mean	St. D.	FP	Р	AFES	SP	Stats (μ JADE (FR) vs μ JADE ε)
C01	μ JADE (FR)	-7.4731E-01	-7.2588E-01	-7.4731E-01	-7.4331E-01	7.4244E-03	1	0.64	55390	86548	
COI	μ JADE ε	-7.4731E-01	-7.2921E-01	-7.4731E-01	-7.4397E-01	5.0826E-03	1	0.64	54301	84845	=
C02	μ JADE (FR)	-7.4095E-01	3.5817E+00	1.8511E+00	1.6164E+00	1.2319E+00	1	0	-	-	
C02	μ JADE ε	-2.2768E+00	-1.2413E+00	-2.2071E+00	-2.1281E+00	2.2529E-01	1	0	-	-	-
C03	μ JADE (FR)	1.3008E+07	1.2412E+15	1.2466E+14	2.4585E+14	3.1525E+14	1	0	-	-	
0.05	μ JADE ε	0.0000E+00	8.8756E+00	4.8227E-24	3.5502E+00	4.4378E+00	1	0.6	38359	63932	-
C04	μ JADE (FR)	-1.0000E-05	1.6189E+01	6.7844E-01	4.4765E+00	6.9197E+00	0.88	0.36	73600	2.04E+05	_
04	μ JADE ε	-1.0000E-05	9.8934E-01	-1.0000E-05	1.7528E-01	3.2354E-01	1	0.72	93651	1.30E+05	-
C05	μ JADE (FR)	1.4418E+02	5.3614E+02	4.2672E+02	4.0444E+02	1.1559E+02	1	0	-	-	
005	μ JADE ε	-4.8361E+02	-4.5982E+02	-4.8361E+02	-4.8060E+02	6.0478E+00	1	0.64	85986	1.34E+05	-
C06	μ JADE (FR)	-5.5580E+01	5.6372E+02	4.0285E+02	3.8466E+02	1.5349E+02	1	0	-	-	_
000	μ JADE ε	-5.7866E+02	-5.7718E+02	-5.7811E+02	-5.7800E+02	6.2002E-01	1	0.2	1.28E+05	6.39E+05	_
C07	μ JADE (FR)	0.0000E+00	3.9866E+00	2.5180E-27	1.5946E-01	7.9732E-01	1	0.96	54094	56348	_
207	μ JADE ε	0.0000E+00	4.9311E-26	2.5180E-27	7.6850E-27	1.1185E-26	1	1	42510	42510	-
C08	μ JADE (FR)	0.0000E+00	4.0876E+01	3.9866E+00	8.2170E+00	1.0210E+01	1	0.48	50476	1.05E+05	_
	μ JADE ε	0.0000E+00	5.0845E+01	1.5541E-26	6.0469E+00	1.1916E+01	1	0.64	35539	55530	
C09	μ JADE (FR)	1.1526E+12	3.1278E+13	8.1924E+12	9.0270E+12	6.1296E+12	1	0	-	-	
	μ JADE ε	0.0000E+00	9.0073E+07	7.7031E-26	3.6038E+06	1.8014E+07	1	0.56	41056	73314	
C10	μ JADE (FR)	1.6163E+12	1.8234E+13	6.7388E+12	7.8844E+12	4.6637E+12	1	0	-	-	_
	μ JADE ε	0.0000E+00	4.8393E+03	4.1728E+01	2.6529E+02	9.6220E+02	1	0.32	70161	2.19E+05	
C11	μ JADE (FR)	-1.5227E-03	-1.3683E-04	-1.5227E-03	-1.4625E-03	2.8898E-04	0.92	0.96	38394	39993	_
	μ JADE ε	-1.5227E-03	-1.5227E-03	-1.5227E-03	-1.5227E-03	3.6696E-09	1	1	69860	69860	
C12	μ JADE (FR)	-3.0549E+02	4.7980E+00	-1.9925E-01	-3.8502E+01	7.6328E+01	1	0.04	70016	1.75E+06	
	μ JADE ε	-4.2652E+02	-1.9924E-01	-1.9925E-01	-1.0413E+02	1.6552E+02	0.96	0	-	-	
C13	μ JADE (FR)	-6.8429E+01	-6.3517E+01	-6.8429E+01	-6.7009E+01	1.7203E+00	1	0.4	1.23E+05	3.07E+05	
	μ JADE ε	-6.8429E+01	-6.5578E+01	-6.5578E+01	-6.6719E+01	1.4253E+00	1	0.32	1.27E+05	3.95E+05	
C14	μ JADE (FR)	0.0000E+00	3.1303E+11	2.7719E+02	1.3609E+10	6.2554E+10	1	0.24	1.20E+05	4.99E+05	-
	μ JADE ε	0.0000E+00	3.9866E+00	4.8878E-27	3.1893E-01	1.1038E+00	1	0.92	46007	50008	
C15	μ JADE (FR)	8.4408E+10	1.6020E+14	4.4344E+13	5.7217E+13	4.6972E+13	1	0	-	-	-
	μ JADE ε	0.0000E+00	1.0607E+01	4.8878E-27	1.3706E+00	2.5952E+00	1	0.68	43658	64203	
C16	μ JADE (FR)	2.1900E-01	1.0394E+00	6.8271E-01	6.5823E-01	2.5065E-01	1	0	-	-	-
	μ JADE ε	0.0000E+00	9.4810E-01	2.9752E-01	3.7532E-01	3.3789E-01	1	0.16	55534	3.47E+05	
C17	μ JADE (FR)	2.5030E+01	9.0098E+02	1.8648E+02	2.4508E+02	2.0868E+02	1	0	-	-	-
	μ JADE ε	1.2326E-32	3.8959E+02	2.6384E-01	5.0345E+01	1.1189E+02	1	0.2	1.19E+05	5.93E+05	
C18	μ JADE (FR)	2.0309E+02	8.2869E+03	3.4694E+03	3.4103E+03	2.0938E+03	1	0	-	-	-
	μ JADE ε	0.0000E+00	3.1554E-30	0.0000E+00	2.5244E-31	8.7370E-31	1	1	26044	26044	

in [14]. The detailed results are shown in Table IV. Table V shows the statistical comparison using the Wilcoxon test, where μ JADE ε was particularly competitive against ε DEag, mainly in 30D test problems. Regarding *DEwAPI*, μ JADE ε was competitive in some test problems, but was outperformed in others, mainly in 30D. As a conclusion of this second experiment, μ JADE ε was competitive, but still not clearly better, than conventional sized populations algorithms for constrained optimization.

C. Third experiment: $\mu JADE\varepsilon$ vs μPSO

Finally, μ JADE ε was compared against one μ EA for constrained spaces: μ PSO [7]. The detailed results are shown in Table VI, where μ JADE ε outperformed μ PSO in ten out of thirteen functions, according to the statistical results. Moreover, the μ JADE ε results were close to the optimal known value. In conclusion, μ JADE ε showed to be highly competitive, compared with other μ EA for constrained optimization.

V. CONCLUSIONS

A highly competitive μ EA for unconstrained optimization called μ JADE was adapted to solve constrained optimization problems. Two constraint-handling techniques, the feasibility rules and the ε -constrained method, were added to μ JADE and compared. The latter provided a better performance based on final statistical results and also based on four performance

TABLE V Statistical comparison among μ JADE ε , ε DEg and DEwAPI using the 95%-confidence Wilcoxon Signed Rank Test. "NA" means that the proposed algorithm did not find feasible solutions.

	μ JADE ε vs.								
Function	1	0D	30D						
	€DEag	DEwAPI	ε DEag	DEwAPI					
C01	-	-	-	=					
C02	-	-	+	-					
C03	-	-	=	-					
C04	+	-	NA	NA					
C05	-	-	-	-					
C06	-	-	-	-					
C07	-	-	=	=					
C08	=	=	=	=					
C09	=	=	=	=					
C10	=	=	=	=					
C11	-	-	=	=					
C12	-	+	-	-					
C13	-	-	+	=					
C14	=	=	=	=					
C15	-	-	=	-					
C16	=	-	=	=					
C17	+	-	+	-					
C18	+	=	+	-					
		Summary							
+	3	1	4	0					
-	10	12	4	8					
=	5	5	9	9					

measures and convergence behavior. μ JADE ε was further compared against two conventionally sized population al-

 TABLE III

 30D results by μ JADE (FR) and μ JADE ε . Bold numbers represent the best values found. "NA" means that the proposed algorithm did not find feasible solutions.

Function	Algorithm	Best	Worst	Median	Mean	St. D.	FP	Р	AFES	SP	Stats (μ JADE (FR)
	"IADE (ED)	8 2122E 01	7 0272E 01	8 1427E 01	8 1200E 01	6 7506E 03	1	0			VS μ JADE ε)
C01	μ JADE (FK)	-8.2083E-01	-7.8861E-01	-8.0987E-01	-8.0980E-01	8.6780E-03	1	0	-	-	=
	μ JADE (FR)	7 3664E-01	3.9577E±00	2 9239E±00	2 9973E±00	7.7112E-01	1	0			-
C02	μ JADE (FR) μ JADE ε	-2.1954E+00	-1.2243E+00	-1.8217E+00	-1.8123E+00	2.8390E-01	1	0.04	4.29E+05	1.07E+07	
	μ JADE (FR)	2.1771E+10	7.9054E+13	3.1189E+13	2.8109E+13	2.5969E+13	0.52	0	-	-	
C03	μ JADE ε	1.8437E-22	1.8321E+02	2.8673E+01	2.7987E+01	3.4643E+01	1	0.64	2.35E+05	3.68E+05	-
C04	µJADE (FR)	-	-	-	-	-	-	0	-	-	NTA
	μ JADE ε	-	-	-	-	-	-	0	-	-	NA
C05	μ JADE (FR)	2.4224E+02	5.5826E+02	4.8759E+02	4.7822E+02	6.5806E+01	1	0	-	-	
0.05	μ JADE ε	-4.1540E+02	5.3688E+02	-2.3891E+02	-7.4365E+01	3.5260E+02	1	0	-	-	-
C06	μ JADE (FR)	2.2947E+02	5.5473E+02	4.8014E+02	4.6503E+02	7.9274E+01	1	0	-	-	-
000	μ JADE ε	-5.2248E+02	-9.4134E-01	-4.7936E+02	-4.0917E+02	1.4767E+02	1	0	-	-	-
C07	μ JADE (FR)	8.7536E-24	2.0070E+01	1.0319E-19	1.6777E+00	4.8985E+00	1	0.84	2.45E+05	2.91E+05	_
	μ JADE ε	3.9874E-24	4.2653E+00	8.3422E-18	2.4312E-01	8.7094E-01	1	0.8	2.68E+05	3.35E+05	_
C08	μ JADE (FR)	1.6567E-23	8.1094E+02	1.3873E-02	5.3416E+01	1.6469E+02	1	0.56	2.80E+05	4.99E+05	=
000	μ JADE ε	1.6493E-23	8.1589E+02	2.7339E-15	6.2209E+01	1.7294E+02	1	0.6	3.02E+05	5.04E+05	
C09	μ JADE (FR)	1.3525E+13	8.0384E+13	3.3298E+13	3.5741E+13	1.4720E+13	1	0	-	-	-
	μ JADE ε	1.1528E+00	5.5671E+10	9.3421E+01	2.2268E+09	1.1134E+10	1	0	-	-	
C10	μ JADE (FR)	1.0306E+13	5.5780E+13	3.3853E+13	3.3608E+13	1.1527E+13	1	0	-	-	-
	μ JADE ε	6.6681E+00	1.9900E+04	4.6657E+01	1.6146E+03	4.6110E+03	1	0	-	-	
C11	μ JADE (FR)	-3.9226E-04	-2.0060E-04	-3.9048E-04	-3.7968E-04	4.3503E-05	0.76	0.88	3.14E+05	3.57E+05	+
	μ JADE ε	-3.9117E-04	1.890/E-04	-3.809/E-04	-2.5862E-04	2.38/9E-04	0.36	0.28	4.02E+05	1.43E+06	
C12	μ JADE (FR)	-1.9926E-01	3.1282E+02	-8.2157E-02	2.1560E+01	6.9655E+01	0.96	0.4	2.10E+05	5.24E+05	+
	μ JADE ε	-1.9291E-01	1.6772E+01	5.5424E-01	3.5009E+00	4.7389E+00	0.72	0	-	-	
C13	µJADE (FK)	-0.0/00E+01	-0.2795E+01	-0.4908E+01	-0.48//E+01	1.1555E+00	1	0	-	-	=
	μ JADE ε	-0.09/5E+01	-0.1539E+01	-0.4093E+01	-0.4551E+01	1.6259E+00	1	0	-	-	
C14	μ JADE (FK)	0.4002E-21	0.7450E+02	4.5105E-11	5.2739E+01	1.4450E+02	1	0.68	3.91E+05	5.74E+05	=
	μ JADE ϵ	0.7215E-25	2.0294E+14	9.1446E-07	2.0035E+00	9 2267E+12	1	0.50	2.37E+03	4.39E+03	
C15	μ JADE (FK)	7 3754F-24	7.6821E±01	1.0210E+14	2.2673E+01	0.5207E+15	1	0.04	- 5.24E±05	- 1.31E±07	-
	μ IADE (FP)	7.8035E.01	1.1456E+00	2.1004E+01	1.0407E±00	8 6080F-02	1	0.04	5.246405	1.516+07	
C16	μ JADE (FK)	0.0000E±00	8 3683F-01	1.0304E+00	3 5330E-02	1.6713E.01	1	0.84	- 16817	20020	-
	μ IADE (FR)	3.8417E+02	1.7673E+03	1.0668E+03	1.0196E+03	3.7742E+02	1	0.04		- 20020	
C17	μ IADE (I R)	3.5857E-02	8.8086E+02	1.6151E+02	1.9562E+02	2.1804E+02	1	0.12	4 37E+05	3.65E+06	-
	μ IADE (FR)	5 6738E+03	5 9899E+04	1 3873E+04	1 5886E+04	1.0906E+04	1	0.12	-	-	
C18	μ IADE (\mathbf{R})	2.2070E-01	1.1974E+04	1.3645E+01	1.9528E+03	3.5428E+03	1	0	-	-	-
					1 200202.00	1	•	l v	1	1	

gorithms, where the results were competitive, but still not clearly better. Moreover, μ JADE ε provided better results when compared against one μ EA for constrained optimization. The overall observed performance provides encouraging insights about the viability of successfully using small populations in DE to deal with constrained search spaces.

As future work, it is necessary to revisit μ JADE ε to analyze its behavior in those problems where local optima were found. The design of mechanisms to increase the exploration abilities with a small population will be studied.

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REFERENCES

- [1] P. Siarry and Z. Michalewicz, Eds., *Advances in Metaheuristics for Hard Optimization*, ser. Natural Computing. Springer, 2008.
- [2] E. Mezura-Montes and C. A. Coello Coello, "Constraint-handling in nature-inspired numerical optimization: Past, present and future," *Swarm* and Evolutionary Computation, vol. 1, no. 4, pp. 173–194, 2011.
- [3] F. Caraffini, F. Neri, and I. Poikolainen, "Micro-differential evolution with extra moves along the axes," in *Differential Evolution (SDE)*, 2013 *IEEE Symposium on*, April 2013, pp. 46–53.

- [4] H. Salehinejad, S. Rahnamayan, H. R. Tizhoosh, and S. Y. Chen, "Micro-differential evolution with vectorized random mutation factor," in *Proceedings of the IEEE Congress on Evolutionary Computation*, *CEC 2014, Beijing, China, July 6-11, 2014*, 2014, pp. 2055–2062. [Online]. Available: http://dx.doi.org/10.1109/CEC.2014.6900606
- [5] C. Brown, Y. Jin, M. Leach, and M. Hodgson, "µ jade: adaptive differential evolution with a small population," *Soft Computing*, pp. 1– 10, 2015.
- [6] X. Yao, Y. Liu, and G. Lin, "Evolutionary programming made faster," *Evolutionary Computation, IEEE Transactions on*, vol. 3, no. 2, pp. 82– 102, Jul 1999.
- [7] J. C. F. Cabrera and C. A. C. Coello, "Handling constraints in particle swarm optimization using a small population size." in *MICAI*, 2007, pp. 41–51.
- [8] T. Runarsson and X. Yao, "Stochastic ranking for constrained evolutionary optimization," *Evolutionary Computation, IEEE Transactions on*, vol. 4, no. 3, pp. 284–294, Sep 2000.
- [9] I. Fajfar, T. Tuma, J. Puhan, J. Olenšek, and Á. Bűrmen, "Towards smaller populations in differential evolution," *Electronic Components* and Materials, vol. 42, no. 3, pp. 152–163, 2012.
- [10] G. T. Pulido and C. A. C. Coello, "A constraint-handling mechanism for particle swarm optimization," in *Evolutionary Computation*, 2004. *CEC2004. Congress on*, vol. 2. Ieee, 2004, pp. 1396–1403.
- [11] T. Takahama and S. Sakai, "Constrained optimization by the epsilon constrained differential evolution with an archive and gradient-based mutation." in *IEEE Congress on Evolutionary Computation*. IEEE, 2010, pp. 1–9.
- [12] R. Mallipeddi and P. Suganthan, "Problem definitions and evaluation criteria for the cec 2010 competition and special session on single objective constrained real-parameter optimization," *Nangyang Technological University, Singapore*, 2010.
- [13] J. Liang, T. P. Runarsson, E. Mezura-Montes, M. Clerc, P. Suganthan, C. C. Coello, and K. Deb, "Problem definitions and evaluation criteria

TABLE IV

Comparison among μ JADE ε , ε DEg and DEWAPI. Bold numbers represent the best values found.

Function Alg 100 source 300 μADEE 7.4731E-01 7.4397E-01 5.0826E-03 8.2083E-01 8.6980E-01 8.6780E-03 C01 ebEag 7.4731E-01 7.4397E-01 5.0826E-03 8.2183E-01 8.2087E-01 7.1039E-04 LDEwAPI 7.4731E-01 7.479E-01 5.12325E-03 8.2183E-01 8.2087E-01 1.1242E-02 LDEwAPI 2.2777E+00 -2.2589E-00 2.3898E-02 2.1692E+00 2.1514E+00 1.1377E-01 C03 eDEag 0.0000E+00 0.0000E+00 0.0000E+00 2.2781E+00 3.472E-01 8.4638E-01 LDEwAPI 1.0000E+00 0.0000E+00 0.0000E+00 3.347E-06 3.347E-06 3.0678E-03 LDEwAPI 1.0000E+05 1.5407E-07 4.9581E-03 3.3207E-06 3.2991E+00 4.5308E+02 3.290E+01 - </th <th>(</th> <th>1</th> <th>1</th> <th>105</th> <th></th> <th>1</th> <th></th> <th></th>	(1	1	105		1		
μADE Jest Mean St. D. Jest Mean St. D. C01 μDEs -7.4731E-01 -7.4704E-01 1.3233E-03 -8.2083E-01 8.0980E-01 8.6780E-03 C02 cbEag -2.2768E+00 -2.1281E+00 -2.2398E-02 -2.1954E+00 -1.8123E+00 2.3390E-01 -2.1954E+00 -2.1514E+00 1.8172E-02 DEwAPI -2.777E+00 -2.2776E+00 -2.2398E-02 -2.1692E+00 -2.278E+00 3.4776E-02 DEwAPI -2.0000E+00 0.0000E+00 0.0000E+00 1.437E-22 -2.778E+00 3.4776E-03 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 3.877E-08 8.687E-08 DEwAPI -0000E-50 0.0000E+00 -3.337E-06 3.237E-06 8.687E-08 DEwAPI -1.0000E-50 0.0000E+00 -3.337E-06 3.236E+01 3.236E+01 3.236E+01 3.237E+06	Function	Alg		10D		D (30D	
μλDEε -7.4731E-01 -7.47970E-01 5.3023E-03 -8.2083E-01 -8.0987E-01 8.1039E-04 C01 gEbag -7.4731E-01 -7.4704E-01 1.3233E-03 8.2183E-01 8.2087E-01 1.1242E-02 μJDE -2.2778E+00 -2.2359E-00 2.3398E-02 -2.1954E+00 1.8123E+00 2.8398E-01 C02 eDewAP1 -2.2777E+00 -2.2376E+00 -6.9912E-04 -2.2371E+00 -3.9470E-03 μJADE -2.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 2.673E+01 2.4632E+01 3.4643E+01 C03 cDeag 0.0000E+00 0.0000E+00 -2.3534E-01 -2.3672E+03 3.2678E-03 DEwAP1 -1.0000E-05 -1.0000E-05 -0.0000E+00 -3.2477E-06 -3.2472E-06 3.2678E-03 DEwAP1 -1.0000E-05 -1.0000E-05 -0.0000E+00 -3.2472E-06 -3.24			Best	Mean	St. D.	Best	Mean	St. D.
C01 EDEage -7.4731E-01 -7.4703E-01 -2.2662E-16 -8.2188E-01 -8.2087E-01 -7.1039E-01 μJADEz -2.2768E+00 -2.2359E+00 -2.21954E+00 -1.8123E+00 2.3390E-01 μJADEz -0.0000E+00 -2.2350E+00 -2.2164E+00 -2.1514E+00 1.7976E-02 μJADEz -0.0000E+00 -0.0000E+00 -0.2389E-02 -2.1632E+00 -2.2777E+00 -3.2478E+01 C03 εDEag -2.2777E+00 -0.22776E+00 -0.2378E+01 -2.8678E+01 -3.2487E+01 C04 eDEag -1.0000E+05 1.07528E-01 3.237E+06 3.067E=03 DEwAPI -1.0000E+05 1.7528E-01 3.237E+06 3.067E=03 3.067E=03 DEwAPI -1.0000E+05 1.0000E+05 0.0300E+00 -2.2348E+02 -2.3297E+00 3.637E+03 JADEE -5.7866E+02 -5.4700E+02 -5.2388E+00 -3.237E+04 3.637E+02 3.2397E+04 JADEE -5.7866E+02 -5.7866E+02 -5.2784E+00 -1.177E+24 2.2438E+02 -2.4991E+01	G01	μ JADE ε	-7.4731E-01	-7.4397E-01	5.0826E-03	-8.2083E-01	-8.0980E-01	8.6780E-03
DEwAPI -7.4731E-01 -7.4731E-01 2.2602E-16 -8.2188E-01 -8.1002-10 1.1242E-00 1.2330E-01 C02 eDEag -2.2777E+00 -2.2389E-00 2.3398E-02 -2.1692E+00 -2.1512+00 1.330E-01 DEwAPI -2.2777E+00 -2.2380E+00 -2.380E+00 -2.380E+00 -2.380E+00 -2.380E+00 -2.380E+00 -3.472E-01 C03 EDEag 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 2.673E+01 -2.4838E+01 8.043E+02 JADEz 1.0000E-05 1.0000E-05 0.0000E+00 -3.347E+06 -3.8487E+08 JJADEz 4.3801E+02 4.8301E+02 4.8301E+02 -4.931E+02 -2.891E+00 DEwAPI 4.3801E+02 4.8301E+02 -4.8301E+02 -4.831E+02 -4.8301E+02	COI	EDEag	-7.4731E-01	-7.4704E-01	1.3233E-03	-8.2183E-01	-8.208/E-01	7.1039E-04
μλDEc -2.1281E+00 -2.1281E+00 -2.1281E+00 -2.1584E+00 -2.27781E+00 -3.9470E-03 μADEE 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 1.8437E=22 2.79871E+01 3.4631E+01 C04 eDEag 0.0000E+00 0.0000E+00 0.26373E+01 2.8384E+01 3.4631E+01 DEwAPI 0.0000E+00 0.0000E+00 -3.247E+06 -3.2497E+06 8.637E+08 DEwAPI 1.0000E+05 1.0000E+05 0.0000E+00 -3.247E+06 -3.2497E+06 8.637E+08 DEwAPI 4.8361E+02 4.8361E+02 4.8361E+02 4.8361E+02 4.8361E+02 4.8361E+02 4.8361E+02 4.8361E+02 4.8361E+02 4.975860+02 5.24881-04 5.23481-01 1.4767E+00 1.23481-01 DEwAPI 4.8361E+02 4.8361E+02 4.8361E+02 4.83781+02 4.273846+01 4.74848-01 1.76826+00		DEWAPI	-7.4731E-01	-7.4731E-01	2.2662E-16	-8.2188E-01	-8.1062E-01	1.1242E-02
C02 εDEage -2.2777E+000 -2.2380E+00 -2.2310E+000 -2.2751E+00 -3.476E+00 01ADEE 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 1.8437E+22 2.7871E+01 3.4643E+01 C03 EDEag 0.0000E+00 0.0000E+00 0.0000E+00 1.2354E-01 2.8573E+01 3.4643E+01 DEwAPI 0.0000E+00 0.0000E+00 1.2770E-24 1.2199E-23 1.1757E-23 C04 z.2534E-01 3.2354E-01 -3.2347E+06 3.8647E+06 8.6837E+08 DEwAPI -1.0000E-05 -1.0000E-05 -0.0000E+00 -4.1540E+02 -7.4355E+01 3.560E+02 DEwAPI -4.3801E+02 -4.8361E+02 -4.8361E+02 <th>G00</th> <th>μJADEε</th> <th>-2.2768E+00</th> <th>-2.1281E+00</th> <th>2.2529E-01</th> <th>-2.1954E+00</th> <th>-1.8123E+00</th> <th>2.8390E-01</th>	G 00	μ JADE ε	-2.2768E+00	-2.1281E+00	2.2529E-01	-2.1954E+00	-1.8123E+00	2.8390E-01
DEwAP1 -2.2776E+00 -2.276E+00 -2.276E+00 -2.2781E+00 -3.9470E+00 C03 μADEs 0.0000E+00 0.0000E+00 0.0000E+00 2.8673E+01 2.8797E+01 3.4643E+01 DEwAP1 0.0000E+00 0.0000E+00 0.0000E+00 1.270E-24 1.2199E-23 1.1757E-23 C04 εDEag -9.923E+06 -9.9185E+06 1.5467E+07 4.6981E+03 3.6678E+03 DEwAP1 -1.0000E+05 1.7528E+00 -4.5430E+02 -4.3501E+02 -3.2497E+06 8.1630E+03 3.560E+03 DEwAP1 -1.0000E+03 0.0000E+00 -4.31490E+02 -4.3551E+02 3.2591E+03 <	C02	EDEag	-2.2777E+00	-2.2589E+00	2.3898E-02	-2.1692E+00	-2.1514E+00	1.1976E-02
μλDEz 0.0000E+00 3.5302E+00 4.4378E+20 2.86738E+01 2.86738E+03 3.0678E+03 C04 εDEag -9.9923E+06 -9.9185E+06 1.5467E+07 4.6981E+03 8.1630E+03 3.0678E+03 μJADEz -1.0000E+05 -1.0000E+05 0.0000E+00 -3.3247E+06 -3.2497E+06 8.6837E-08 DEwAPI -1.0000E+05 -1.0000E+05 -1.6038E+13 -4.5313E+02 -4.8361E+02 -2.8991E+00 DEwAPI -4.8361E+02 -4.8361E+02 -4.8361E+02 -4.8361E+02 -4.8361E+02 -2.8786E+02 -5.2788E+01 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5288E+02 -5.2788E+01 -2.6529E+02 -2.7911E+02 -4.7438E+01 DEwAPI -5.7866E+02 -5.7866E+02 -5.7866E+02 -5.858E+01 -1.786E+00 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5288E+01 -2.5282		DEWAPI	-2.2777E+00	-2.2776E+00	6.9912E-04	-2.2810E+00	-2.2/51E+00	3.9470E-03
C03 εDExAPI 0.0000E+00 0.0000E+00 0.0000E+00 1.23673E-24 1.239E-13 1.1757E-23 C04 zDEag 9.993E-06 1.5407E-07 4.6981E-03 8.1630E-03 3.0678E-03 DEwAPI -1.0000E-05 -1.0000E-05 -0.000E+00 -3.2347E-06 -3.2437E-06 8.6837E-08 DEwAPI -1.3000E-05 -1.0000E-05 -0.000E+00 -3.2347E-06 -3.2437E-06 8.6837E-08 C05 EDEag -4.3501E+02 -4.8301E+02 -4.8301E+02 -4.8301E+02 -4.4351E+02 -4.4955E+02 -2.2971E+02 -2.7476E-01 DEwAPI -5.7866E+02 -5.7866E+02 -5.7866E+02 -5.2385E+02 -5.2385E+02 -1.2771E-0 C06 zDEwAPI -0.0000E+00 -7.685E+02 -5.7866E+02 -5.7866E+02 -5.303E+02 -3.2371E-03 -5.22858E+02 -2.303E+02 1.178E+00 C06 zDEwAPI -0.0000E+00 0.0000E+00 0.0000E+00 0.1477E+02 -2.333E+02 -2.333E+02 -2.333E+02 -2.333E+01 1.7294E+02 C07 <th>~~~</th> <th>μJADEε</th> <th>0.0000E+00</th> <th>3.5502E+00</th> <th>4.4378E+00</th> <th>1.8437E-22</th> <th>2.7987E+01</th> <th>3.4643E+01</th>	~~~	μ JADE ε	0.0000E+00	3.5502E+00	4.4378E+00	1.8437E-22	2.7987E+01	3.4643E+01
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	C03	εDEag	0.0000E+00	0.0000E+00	0.0000E+00	2.8673E+01	2.8838E+01	8.0472E-01
μ/ADE -1.0000E-05 1.7528E-01 3.2534E-01 -		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	1.2770E-24	1.2199E-23	1.1757E-23
C04 εDEaxpl -0.9923E-06 -9.9185E-06 1.3467E-07 4.6981E-03 8.1530E-03 3.30678E-08 μIADEE -4.8361E+02 -4.8361E+02 -6.978E+00 -3.2477E-06 -3.2477E-06 3.2497E-06 3.2497E-01 3.52560E+02 2.5806E+02 3.500E+02 3.5291E+01 1.4761E+02 1.4761E+02 1.4761E+02 4.4761E+02 4.4851E+04 4.4852E+14		μ JADE ε	-1.0000E-05	1.7528E-01	3.2354E-01	-	-	-
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	C04	ε DEag	-9.9923E-06	-9.9185E-06	1.5467E-07	4.6981E-03	8.1630E-03	3.0678E-03
μλΔΕε -4.350E+02 -4.360E+02 -6.0478E+00 -4.1540E+02 -7.4365E+01 3.5200E+02 C05 cDEwAPI -4.356E+02 -4.356E+02 -4.356E+02 -2.899E+00 DEwAPI -4.356E+02 -5.7866E+02 -5.7800E+02 -5.2248E+02 -5.279E+02 -4.7435E+02 -2.579E+02 -4.7436E+02 C06 cDEwAPI -5.7866E+02 -5.7866E+02 -5.2786E+02 -5.2388E+02 -5.2303E+02 1.1786E+00 DEwAPI -5.7866E+02 -5.7866E+02 -5.3064E+02 -5.303E+02 1.1786E+00 C07 cDEwAPI 0.0000E+00 0.0000E+00 0.11471E+15 2.6036E+15 1.2334E+15 DEwAPI 0.0000E+00 0.0000E+00 1.1471E+15 2.6036E+15 1.2334E+15 DEwAPI 0.0000E+00 0.3638E+06 1.15128E+00 2.5228E+02 6.208E+04 1.134E+10 DEwAPI 0.0000E+00 0.36038E+06 1.304E+07 1.1528E+00 2.2268E+09 1.134E+10 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00		DEwAPI	-1.0000E-05	-1.0000E-05	0.0000E+00	-3.3247E-06	-3.2497E-06	8.6837E-08
C05 cDEage 4.3361E+02 4.38361E+02 4.3361E+02 4.4955E+02 2.8991E+00 DEwAPI 4.4361E+02 4.4361E+02 4.361E+02 4.361E+02 4.3561E+02 4.3561E+02 4.3561E+02 4.3561E+02 4.3561E+02 4.3561E+02 4.7855E+02 3.6272E+03 5.2838E+02 5.2791E+02 4.7484E-01 DEwAPI 5.7866E+02 5.7865E+02 1.8415E+08 5.3064E+02 5.3082+02 1.736E+00 C07 cDEag 0.0000E+00 0.0000E+00 0.0000E+00 1.6140E-26 1.1662E-25 7.1627E-26 DEwAPI 0.0000E+00 6.0469E+00 1.1916E+01 1.6433E-23 6.2209E+01 1.7294E+02 C08 cDEag 0.0000E+00 3.6038E+06 3.1361E+00 8.9686E-26 5.3824E+00 2.4456E+01 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.2767E+10 1.5327E+14 7.8373E-23 µJADEz 0.0000E+00 0.0000E+00 0.0000E+00 1.7707E+16 1.7721E+01 2.8219E+01 DEwAPI 0.0000E+00		μ JADE ε	-4.8361E+02	-4.8060E+02	6.0478E+00	-4.1540E+02	-7.4365E+01	3.5260E+02
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	C05	ε DEag	-4.8361E+02	-4.8361E+02	3.8904E-13	-4.5313E+02	-4.4955E+02	2.8991E+00
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	-4.8361E+02	-4.8361E+02	1.1603E-13	-4.8361E+02	-4.8361E+02	2.6559E-03
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		μ JADE ε	-5.7866E+02	-5.7800E+02	6.2002E-01	-5.2248E+02	-4.0917E+02	1.4767E+02
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	C06	ε DEag	-5.7866E+02	-5.7865E+02	3.6272E-03	-5.2858E+02	-5.2791E+02	4.7484E-01
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	-5.7866E+02	-5.7866E+02	1.8415E-08	-5.3064E+02	-5.3033E+02	1.1786E+00
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		μ JADE ε	0.0000E+00	7.6850E-27	1.1185E-26	3.9874E-24	2.4312E-01	8.7094E-01
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	C07	ε DEag	0.0000E+00	0.0000E+00	0.0000E+00	1.1471E-15	2.6036E-15	1.2334E-15
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	1.6140E-26	1.1662E-25	7.1627E-26
C08 εDEag 0.0000E+00 6.7275E+00 5.5606E+00 2.5187E-14 7.8315E-14 4.8552E-14 DEwAPI 0.0000E+00 3.0361E+00 3.1361E+00 8.9686E-26 5.3824E+00 2.4456E+01 C09 εDEag 0.0000E+00 3.0363E+00 1.8014E+07 1.1528E+00 2.2268E+09 1.1134E+10 DEwAPI 0.0000E+00 0.000E+00 0.0000E+00 2.6529E+02 6.6681E+00 1.6343E-23 7.8733E-23 µJADEE 0.0000E+00 0.0000E+00 0.0000E+00 3.6596E-09 3.3262E+01 4.5456E-01 DEwAPI 0.0000E+00 0.0000E+00 0.3250E+02 3.3262E+01 4.5456E-01 JADEE -1.5227E-03 -1.5227E-03 6.3410E+11 -3.2685E-04 -2.8639E-04 2.3879E-04 µJADE -1.5227E-03 -1.5227E-03 6.3410E+11 -3.2384E+04 -3.3234E+04 2.3736E-09 DEwAPI -5.237E+02 -1.9271E-01 -3.509E+00 4.7338E+00 -6.6322E+01 -6.6435E+01 -1.9226E-01 1.7338E+00 C12 <		μ JADE ε	0.0000E+00	6.0469E+00	1.1916E+01	1.6493E-23	6.2209E+01	1.7294E+02
DEwAPI 0.0000E+00 1.8075E+00 3.1361E+00 8.9686E-26 5.3824E+00 2.4456E+01 C09 zDEag 0.0000E+00 0.0000E+00 0.0000E+00 2.2088E+09 1.1134E+10 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 2.707E+16 1.221E+01 2.23819E+01 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.27707E+16 1.6146E+03 4.6110E+03 C10 zDEag 0.0000E+00 0.0000E+00 0.0000E+00 3.2520E+01 3.3262E+01 4.5456E-01 DEwAPI 0.0000E+00 0.0000E+00 5.7761E-26 2.0463E-24 5.3676E-24 µJADEE -1.5227E-03 -1.5227E-03 5.9927E-10 -3.917E-04 -2.5862E-04 2.3879E-04 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9034E-04 2.7766-05 DEwAPI -3.0549E+02 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9234E-04 2.7736E-09 DEwAPI -3.0549E+02 -1.5227E-03 5.9927E-10 -3.5029E-01	C08	ε DEag	0.0000E+00	6.7275E+00	5.5606E+00	2.5187E-14	7.8315E-14	4.8552E-14
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	0.0000E+00	1.8075E+00	3.1361E+00	8.9686E-26	5.3824E+00	2.4456E+01
C09 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 2.7707E-16 1.0721E+01 2.8219E+01 μADEε 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 3.2520E+01 1.6343E-23 7.8733E-23 C10 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 3.2520E+01 3.3262E+01 4.5456E-01 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 5.7761E-26 2.0463E-24 5.3676E-24 μJADEε -1.5227E-03 -1.5227E-03 3.6696E-09 -3.9117E-04 -2.5862E-04 2.839E-04 2.776E-05 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9234E-04 2.7736E-09 L1 εDEag -5.709E+02 -2.4554E+01 1.6552E+02 -1.921E-01 3.5623E+00 2.8893E+02 DEwAPI -3.0549E+02 -2.4554E+01 1.4253E+00 -6.6425E+01 -6.4333E+01 1.7433E-06 DEwAPI -3.0549E+04 -6.8429E+01 1.4253E+09 -6.6362E+01 -6.4345E+01 1.6235E+00 <th></th> <th>μJADEε</th> <th>0.0000E+00</th> <th>3.6038E+06</th> <th>1.8014E+07</th> <th>1.1528E+00</th> <th>2.2268E+09</th> <th>1.1134E+10</th>		μ JADE ε	0.0000E+00	3.6038E+06	1.8014E+07	1.1528E+00	2.2268E+09	1.1134E+10
DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 1.6343E-23 7.8733E-23 μJADEε 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.620E+00 3.2520E+01 3.362E+01 4.6110E+03 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 3.2520E+01 3.3262E+01 4.5456E-01 DEwAPI -1.5227E-03 -1.5227E-03 3.6696E-09 -3.9117E-04 -2.5862E-04 2.3879E-04 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E+10 -3.9234E-04 -3.9234E-04 2.7076E-05 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E+10 -3.9234E-04 -2.8639E-04 2.7076E-05 C12 εDEag -5.7009E+02 -3.3673E+02 1.7822E+02 -1.9921E-01 3.5028E+00 4.7338E+00 C13 εDEag -6.8429E+01 -6.6719E+01 1.4253E+00 -6.6975E+01 -6.4531E+01 1.6259E+00 C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E-1	C09	εDEag	0.0000E+00	0.0000E+00	0.0000E+00	2.7707E-16	1.0721E+01	2.8219E+01
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.6343E-23	7.8733E-23
C10 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 3.2520E+01 3.3262E+01 4.5456E-01 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 5.7761E-26 2.0463E-24 5.3676E-24 µJADEz -1.5227E-03 -1.5227E-03 3.6696E-09 -3.9117E-04 -2.5862E-04 2.3879E-04 C11 eDEag -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -2.8639E-04 2.7076E-05 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9234E-04 2.8893E+02 cDewAPI -4.2652E+02 -1.0413E+02 1.6552E+02 -1.9926E-01 1.7432E-06 1.9926E-01 1.7432E-06 DEwAPI -6.8429E+01 -6.6719E+01 1.4253E+00 -6.6975E+01 -6.5353E+01 5.7330E-01 DEwAPI -6.8429E+01 -6.8429E+01 5.5258E-09 -6.6362E+01 -6.4343E+01 1.1404E+00 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E+13 DEwAPI 0.0000E	C10	μ JADE ε	0.0000E+00	2.6529E+02	9.6220E+02	6.6681E+00	1.6146E+03	4.6110E+03
DEwAPI 0.0000E+00 0.0000E+00 5.7761E-26 2.0463E-24 5.3676E-24 μJADEε -1.5227E-03 -1.5227E-03 3.6696E-09 -3.9117E-04 -2.5862E-04 2.3879E-04 C11 εDEag -1.5227E-03 -1.5227E-03 5.9927E-10 -3.2685E-04 -2.8639E-04 2.7736E-09 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9234E-04 2.7736E-09 μJADEε -4.2652E+02 -1.0413E+02 1.6552E+02 -1.9915E-01 3.5009E+00 4.7389E+00 C12 εDEwAPI -3.0549E+02 -2.4554E+01 8.4295E+01 -1.9926E-01 -1.9926E-01 1.7432E-06 C13 εDEag -6.8429E+01 -6.6719E+01 1.0206E-06 -6.6425E+01 -6.533E+01 5.7330E-01 DEwAPI -6.8429E+01 -6.8429E+01 5.5258E-09 -6.632E+01 -6.3438E+01 1.1404E+00 C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E-13 DEwAPI 0.0000E+00		εDEag	0.0000E+00	0.0000E+00	0.0000E+00	3.2520E+01	3.3262E+01	4.5456E-01
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	5.7761E-26	2.0463E-24	5.3676E-24
C11 ε DEag -1.5227E-03 -1.5227E-03 5.9927E-10 -3.2685E-04 -2.8639E-04 2.7076E-05 DEwAPI -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9234E-04 2.7736E-09 µJADEε -4.2652E+02 -1.0413E+02 1.6552E+02 -1.991E-01 3.5009E+00 4.7389E+00 C12 εDEag -5.7009E+02 -3.3673E+02 1.7822E+02 -1.991E-01 3.5023E+02 2.893E+02 2.893E+02 DEwAPI -3.0549E+01 -6.6719E+01 1.4253E+00 -6.6975E+01 -6.4531E+01 1.6259E+00 c13 εDEag 6.8429E+01 -6.8429E+01 5.528E+09 -6.6362E+01 -6.433E+01 1.140E+00 μJADEε 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E-13 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 7.3752E-01 7.1632E-24 2.2673E+01 7.6121E+01 L1404E±01 1.3706E+00 2.5952E+00 7.3538E-26 1.5947E-01 7.9732E-01 7.1603E+01 7.1042E+		μ JADE ε	-1.5227E-03	-1.5227E-03	3.6696E-09	-3.9117E-04	-2.5862E-04	2.3879E-04
DEwAPI -1.5227E-03 -1.5227E-03 5.9927E-10 -3.9234E-04 -3.9234E-04 2.7736E-09 μJADEε -4.2652E+02 -1.0413E+02 1.6552E+02 -1.9291E-01 3.5009E+00 4.7389E+00 C12 εDEag -5.7009E+02 -3.3673E+02 1.7822E+02 -1.9915E-01 3.5623E+02 2.8893E+02 DEwAPI -3.0549E+02 -2.4554E+01 8.4295E+01 -1.9926E-01 -1.9926E-01 1.7433E-06 DEwAPI -6.8429E+01 -6.8429E+01 1.4253E+00 -6.6755E+01 -6.4531E+01 1.6259E+00 DEwAPI -6.8429E+01 -6.8429E+01 5.5258E-09 -6.6362E+01 -6.433E+01 5.7330E-01 µJADEε 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E+13 DEwAPI 0.0000E+00 0.3000E+00 0.0000E+00 7.3754E-24 2.2673E+01 1.5712E+01 JADEε 0.0000E+00 1.3705E+01 8.132E+01 2.1603E+01 2.1604E+01 1.5771E+00 C16 εDEag 0.0000E+00 3.7532E+01 </th <th>C11</th> <th>€DEag</th> <th>-1.5227E-03</th> <th>-1.5227E-03</th> <th>6.3410E-11</th> <th>-3.2685E-04</th> <th>-2.8639E-04</th> <th>2.7076E-05</th>	C11	€DEag	-1.5227E-03	-1.5227E-03	6.3410E-11	-3.2685E-04	-2.8639E-04	2.7076E-05
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	-1.5227E-03	-1.5227E-03	5.9927E-10	-3.9234E-04	-3.9234E-04	2.7736E-09
C12 ε DEag -5.7009E+02 -3.3673E+02 1.7822E+02 -1.9915E-01 3.5623E+02 2.8893E+02 DEwAPI -3.0549E+02 -2.4554E+01 8.4295E+01 -1.9926E-01 -1.9926E-01 1.7433E-06 C13 εDEag -6.8429E+01 -6.6719E+01 1.4253E+00 -6.6325E+01 -6.4531E+01 1.6259E+00 DEwAPI -6.8429E+01 -6.8429E+01 5.5258E-09 -6.6362E+01 -6.4343E+01 1.1404E+00 DEwAPI -6.8429E+01 5.5258E-09 -6.6362E+01 -6.330E+01 5.7330E-01 C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E-13 DEwAPI 0.0000E+00 0.0000E+00 1.5033E-26 1.5947E-01 7.9732E-01 JADEε 0.0000E+00 1.3706E+00 2.5952E+00 7.354E-24 2.2673E+01 1.6213E+01 cEbag 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E-02 1.6713E+01 L625220 DEwAPI 0.0000E+00 3.702E-11		μ JADE ε	-4.2652E+02	-1.0413E+02	1.6552E+02	-1.9291E-01	3.5009E+00	4.7389E+00
DEwAPI -3.0549E+02 -2.4554E+01 8.4295E+01 -1.9926E-01 -1.9926E-01 1.7433E-06 μJADEε -6.8429E+01 -6.6719E+01 1.4253E+00 -6.6775E+01 -6.4531E+01 1.6259E+00 C13 εDEag -6.8429E+01 -6.8429E+01 1.0260E+06 -6.6325E+01 -6.4531E+01 1.16259E+00 DEwAPI -6.8429E+01 -6.8429E+01 5.5258E+09 -6.6362E+01 -6.3535E+00 5.3917E+00 C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E+14 3.0894E+13 5.6084E+13 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 7.3754E-24 2.2673E+01 1.1048E+00 eDEag 0.0000E+00 1.7306E+00 2.5952E+00 7.3754E-24 2.2673E+01 1.6118E+01 eDEag 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E+02 1.6713E+01 eDEwAPI 0.0000E+00 3.702E+01 3.7158E+01 0.0000E+00 3.6000E+00 3.6000E+00 3.6000E+00 3.60000E+00 0.0000E+00 0.	C12	€DEag	-5.7009E+02	-3.3673E+02	1.7822E+02	-1.9915E-01	3.5623E+02	2.8893E+02
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	-3.0549E+02	-2.4554E+01	8.4295E+01	-1.9926E-01	-1.9926E-01	1.7433E-06
C13 ε DEag -6.8429E+01 -6.8429E+01 1.0260E-06 -6.6425E+01 -6.5353E+01 5.7330E-01 DEwAPI -6.8429E+01 -6.8429E+01 5.5258E-09 -6.6362E+01 -6.4343E+01 1.1404E+00 µJADEz 0.0000E+00 3.1893E-01 1.1038E+00 8.7215E-25 2.0635E+00 5.3917E+00 C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E-13 DEwAPI 0.0000E+00 1.3706E+00 2.5952E+00 7.3754E-24 2.2673E+01 1.6213E+01 cEbag 0.0000E+00 1.7909E-01 8.8132E-01 2.1604E+01 1.1071E+01 DEwAPI 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E-02 1.6713E-01 L5771E+00 0.0000E+00 3.7021E-01 3.7105E-01 0.0000E+00 3.5339E-02 1.6713E-01 cDewAPI 0.0000E+00 3.7021E-01 3.7189E-01 0.0000E+00 3.6000E+00 0.0000E+00 0.0000E+00 3.63537E+02 1.9562E+02 1.1648E+21 1.062		μ JADE ε	-6.8429E+01	-6.6719E+01	1.4253E+00	-6.6975E+01	-6.4531E+01	1.6259E+00
DEwAPI -6.8429E+01 -6.8429E+01 5.5258E-09 -6.6362E+01 -6.4343E+01 1.1404E+00 μJADEε 0.0000E+00 3.1893E-01 1.1038E+00 8.7215E-25 2.0635E+00 5.3917E+00 C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E+13 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 1.5033E-26 1.5947E-01 7.9732E-01 μJADEε 0.0000E+00 1.3706E+00 2.5952E+00 7.3754E-24 2.2673E+01 1.6213E+01 C15 εDEag 0.0000E+00 3.732E-01 3.3789E-01 2.1603E+01 2.1604E+01 1.5771E+00 DEwAPI 0.0000E+00 3.7021E-01 3.715E-01 0.0000E+00 2.16328-01 0.0000E+00 2.16328-01 C16 εDEag 0.0000E+00 3.7021E-01 3.715E-01 0.0000E+00 2.1684E-21 1.0623E-20 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 2.1684E-121 1.0623E-20 DEwAP	C13	εDEag	-6.8429E+01	-6.8429E+01	1.0260E-06	-6.6425E+01	-6.5353E+01	5.7330E-01
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	-6.8429E+01	-6.8429E+01	5.5258E-09	-6.6362E+01	-6.4343E+01	1.1404E+00
C14 εDEag 0.0000E+00 0.0000E+00 0.0000E+00 5.0159E-14 3.0894E-13 5.6084E-13 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 1.5032E-26 1.5947E-01 7.9732E-01 µJADEε 0.0000E+00 1.3706E+00 2.5952E+00 7.3754E-24 2.2673E+01 1.6213E+01 C15 εDEag 0.0000E+00 1.7900E-01 8.8132E-01 2.1603E+01 2.1604E+01 1.1048E-04 DEwAPI 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E-02 1.6713E-01 C16 εDEag 0.0000E+00 3.7021E-01 3.7105E-01 0.0000E+00 3.5339E-02 1.6713E-01 C16 εDEwAPI 0.0000E+00 3.7021E-01 3.7105E-01 0.0000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.0000E+00 0.0000E+00 0.0000E+00		μ JADE ε	0.0000E+00	3.1893E-01	1.1038E+00	8.7215E-25	2.0635E+00	5.3917E+00
DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 1.5033E-26 1.5947E-01 7.9732E-01 μJADEε 0.0000E+00 1.3706E+00 2.5952E+00 7.3754E-24 2.2673E+01 1.6213E+01 c15 εDEag 0.0000E+00 1.7990E-01 8.8132E-01 2.1603E+01 2.1604E+01 1.16213E+01 DEwAPI 0.0000E+00 1.6315E+15 8.1577E+15 7.4342E-27 6.7441E-01 1.5771E+00 μJADEε 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E-02 1.6713E-01 DEwAPI 0.0000E+00 3.7021E-01 3.7105E-01 0.0000E+00 2.1684E-21 1.0623E-20 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 μJADEε 1.2326E-32 5.0345E+01 1.1189E+02 3.5857E-02 1.9562E+02 2.1804E+02 EWAPI 0.0000E+00 3.4513E-33 3.1223E-33 1.4385E+13 1.4861E-02 1.3969E-02 DEwAPI 0.0000E+00 2.5244E-31 8.7370E-31	C14	εDEag	0.0000E+00	0.0000E+00	0.0000E+00	5.0159E-14	3.0894E-13	5.6084E-13
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	1.5033E-26	1.5947E-01	7.9732E-01
C15 ε DEag DEwAPI 0.0000E+00 1.7990E-01 8.8132E-01 2.1603E+01 2.1604E+01 1.1048E-04 μJADEε 0.0000E+00 1.6315E-15 8.1577E-15 7.4342E-27 6.7441E-01 1.5771E+00 μJADEε 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E-02 1.6713E-01 c16 εDEag 0.0000E+00 3.7021E-01 3.7105E-01 0.0000E+00 0.000E+00		μ JADE ε	0.0000E+00	1.3706E+00	2.5952E+00	7.3754E-24	2.2673E+01	1.6213E+01
DEwAPI 0.0000E+00 1.6315E-15 8.1577E-15 7.4342E-27 6.7441E-01 1.5771E+00 μJADEε 0.0000E+00 3.7532E-01 3.3789E-01 0.0000E+00 3.5339E-02 1.6713E-01 C16 εDEag 0.0000E+00 3.7021E-01 3.7105E-01 0.0000E+00 2.1684E-21 1.0623E-20 DEwAPI 0.0000E+00 0.000E+00	C15	εDEag	0.0000E+00	1.7990E-01	8.8132E-01	2.1603E+01	2.1604E+01	1.1048E-04
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		DEwAPI	0.0000E+00	1.6315E-15	8.1577E-15	7.4342E-27	6.7441E-01	1.5771E+00
C16 ε DEag DEwAPI 0.000E+00 3.7021E-01 3.7105E-01 0.0000E+00 2.1684E-21 1.0623E-20 μJADEε 1.2326E-32 5.0345E+01 1.1189E+02 3.5857E+02 1.9562E+02 2.1804E+02 c17 εDEag 1.4632E-17 1.2496E+01 1.9372E+01 2.1657E+01 6.3265E+00 4.9867E+00 DEwAPI 0.0000E+00 3.4513E-33 3.1223E-33 1.4885E-13 1.4861E-02 1.3969E+02 μJADEε 0.0000E+00 3.4513E-33 3.1223E-33 1.4385E-13 1.4861E-02 1.3969E+02 μJADE 0.0000E+00 3.4513E-33 3.1223E+33 1.4261E+02 3.5426E+01 cE0Eag 3.7314E-20 9.5788E+19 1.8112E+18 1.2261E+00 8.7546E+01 1.6648E+02 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 9.4958E+12 3.6518E-01 1.7345E+00		μ JADE ε	0.0000E+00	3.7532E-01	3.3789E-01	0.0000E+00	3.5339E-02	1.6713E-01
DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 μJADEε 1.2326E-32 5.0345E+01 1.1189E+02 3.5857E-02 1.9562E+02 2.1804E+02 cDEag 1.4632E-17 1.2496E-01 1.9372E-01 2.1657E-01 6.3265E+00 4.9867E+00 DEwAPI 0.0000E+00 3.4513E-33 3.1223E-33 1.4385E-13 1.4861E-02 1.3969E-02 μJADEε 0.0000E+00 2.5244E-31 8.7370E-31 2.2070E-01 1.9528E+03 3.5428E+03 cDEag 3.7314E-20 9.6788E-19 1.8112E-18 1.2261E+00 8.7546E+01 1.6648E+02 DEwAPI 0.0000E+00 0.0000E+00 9.0900E+00 9.4958E-12 3.6518E-01 1.7345E+00	C16	εDEag	0.0000E+00	3.7021E-01	3.7105E-01	0.0000E+00	2.1684E-21	1.0623E-20
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
C17 ε DEag 1.4632E-17 1.2496E-01 1.9372E-01 2.1657E-01 6.3265E+00 4.9867E+00 DEwAPI 0.0000E+00 3.4513E-33 3.1223E-33 1.4385E-13 1.4861E-02 1.3969E-02 µJADEε 0.0000E+00 2.5244E-31 8. 7370E-31 2.2070E-01 1.9528E+03 3.5428E+03 c18 εDEag 3.7314E-20 9.6788E+19 1.8112E-18 1.2261E+00 8.7546E+01 1.6648E+02 DEwAPI 0.0000E+00 0.0000E+00 9.4958E+12 3.6518E-01 1.7345E+00		μ JADE ε	1.2326E-32	5.0345E+01	1.1189E+02	3.5857E-02	1.9562E+02	2.1804E+02
DEwAPI 0.0000E+00 3.4513E-33 3.1223E-33 1.4385E-13 1.4861E-02 1.3969E-02 μJADEε 0.0000E+00 2.5244E-31 8.7370E-31 2.2070E-01 1.9528E+03 3.5428E+03 c18 εDEag 3.7314E-20 9.6788E-19 1.8112E-18 1.2261E+00 8.7546E+01 1.6648E+02 DEwAPI 0.0000E+00 0.0000E+00 9.0000E+00 9.4958E+12 3.6518E-01 1.7345E+00	C17	€DEag	1.4632E-17	1.2496E-01	1.9372E-01	2.1657E-01	6.3265E+00	4.9867E+00
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		DEwAPI	0.0000E+00	3.4513E-33	3.1223E-33	1.4385E-13	1.4861E-02	1.3969E-02
C18 εDEag 3.7314E-20 9.6788E-19 1.8112E-18 1.2261E+00 8.7546E+01 1.6648E+02 DEwAPI 0.0000E+00 0.0000E+00 0.0000E+00 9.4958E-12 3.6518E-01 1.7345E+00		μ JADE ε	0.0000E+00	2.5244E-31	8.7370E-31	2.2070E-01	1.9528E+03	3.5428E+03
DEWAPI 0.0000E+00 0.0000E+00 0.0000E+00 9.4958E-12 3.6518E-01 1.7345E+00	C18	€DEag	3.7314E-20	9.6788E-19	1.8112E-18	1.2261E+00	8.7546E+01	1.6648E+02
		DEwAPI	0.0000E+00	0.0000E+00	0.0000E+00	9.4958E-12	3.6518E-01	1.7345E+00

for the cec 2006 special session on constrained real-parameter optimiza-tion," *Journal of Applied Mechanics*, vol. 41, p. 8, 2006.

- [14] S. M. Elsayed, R. Sarker *et al.*, "Differential evolution with automatic population injection scheme for constrained problems," in *Differential Evolution (SDE), 2013 IEEE Symposium on.* IEEE, 2013, pp. 112–118.
 [15] E. Mezura-Montes, M. E. Miranda-Varela, and R. del Carmen Gómez-Ramón, "Differential evolution in constrained numerical optimization: provide trade;" *Locarce and Evolution Science and Provide Tradematical Constrained Con*
- an empirical study," Information Sciences, vol. 180, no. 22, pp. 4223-4262, 2010.

TABLE VI Statistical comparison between μ JADE ε , and μ PSO using the 95%-confidence Wilcoxon Signed Rank Test on 13 test problems.

Function	Optimal	Algorithm	Best	Worst	Median	Mean	St. D.	Stats (μ JADE ε vs μ PSO)	
c01 15		μ JADE ε	-15	-15	-15	-15	2.54E-16		
gui	-15	μPSO	-15	-13.273	-13	-9.7012	1.41	+	
g02	0.903610104	μ JADE ε	-0.8036	-0.7535	-0.7926	-0.79298	0.011143		
	-0.803019104	μPSO	-0.80362	-0.77714	-0.77848	-0.7116	0.0191	-	
~03	1 0005001	μ JADE ε	-1.0005	-1.0003	-1.0005	-1.0005	3.61E-05		
gus	-1.0005001	μPSO	-1.0004	-0.9936	-1.0004	-0.6674	0.0471	+	
~04	20665 52867	μ JADE ε	-30666	-30666	-30666	-30666	1.23E-11		
g04	-30003.33607	μPSO	-30666	-30666	-30666	-30666	0.000683	-	
a05	5126 406714	μ JADE ε	5126.5	5126.5	5126.5	5126.5	4.20E-12		
gus	5120.490/14	μPSO	5126.6	5495.2	5261.8	6272.7	405	+	
a06	-6961.813876	μ JADE ε	-6961.8	-6961.8	-6961.8	-6961.8	4.59E-12	+	
guu		μPSO	-6961.8	-6961.8	-6961.8	-6961.8	0.000261		
~07	24.30620907	μ JADE ε	24.307	24.557	24.328	24.354	0.05834	+	
gu/		μPSO	24.328	24.7	24.645	25.296	0.252		
a08	-0.095825042	μ JADE ε	-0.095825	-0.095825	-0.095825	-0.095825	8.41E-17		
guo		μPSO	-0.095825	-0.095825	-0.095825	-0.095825	0	-	
a00	680 6300574	μ JADE ε	680.63	680.63	680.63	680.63	0.00057886		
guy	000.0300374	μPSO	680.63	680.64	680.64	680.67	0.00668	-	
a10	7040 248021	μ JADE ε	7049.3	7251	7060.7	7100.9	70.704		
giu	7047.240021	μPSO	7090.5	7747.6	7557.4	10534	552	+	
g11	0 7400	μ JADE ε	0.7499	0.7499	0.7499	0.7499	3.36E-16		
	0.7433	μPSO	0.7499	0.7673	0.7499	0.9925	0.06	-	
~12	1	μ JADE ε	-1	-1	-1	-1	0	_	
g12	-1	μPSO	-1	-1	-1	-1	0		
σ13	0.0530/151/	μ JADE ε	0.053942	1	0.053942	0.10365	0.16684	+	
gi5	0.055941514	μPSO	0.05941	0.81335	0.90953	2.4442	0.381	+	