MODE-LD+SS: A Novel Differential Evolution Algorithm Incorporating Local Dominance and Scalar Selection Mechanisms for Multi-Objective Optimization

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***Abstract*— In this paper, we present a novel Multi-Objective Evolutionary Algorithm (MOEA) called MODE-LD+SS, which combines Differential Evolution with local dominance and a scalar selection mechanism for improving both its convergence rate and its distribution of solutions along the Pareto front. In order to assess the performance of the proposed approach, we use a set of standard test functions and performance measures taken from the specialized literature. Results are compared with respect to two MOEAs representative of the state-of-the-art in the area: NSGA-II and SPEA2.**

I. Introduction

Many real-world optimization problems require the simultaneous optimization of two or more objective functions. Such problems are called Multi-Objective Optimization Problems (MOPs). In contrast with single-objective optimization problems, MOPs do not have a single solution, but a set of them, which correspond to the best possible trade-offs among the objectives (i.e., no further improvement in one objective is possible without worsening another one). These solutions are contained in the so-called *Pareto optimal set* (the vectors of the solutions contained in the Pareto optimal set are called *nondominated*) and their corresponding objective function values are called the *Pareto front*. MOPs have been a subject of study within Operations Research for several years [17], but the limitations of mathematical programming techniques have motivated the use of evolutionary algorithms to solve them. Multi-objective evolutionary algorithms (MOEAs) have gained popularity mainly because of their generality (i.e., they require little problem-specific information), ease of use and effectivity. A wide variety of MOEAs are currently available, although few of them have become popular [5]. MOEAs aim to find solutions that are as close as possible to the true Pareto front but that, at the same time, are as diverse as possible, so that the entire Pareto front can be covered. These two goals turn out to be quite difficult in some cases and has motivated a significant amount of research. Here, we present a MOEA called MODE-LD+SS, which is based on the use of Differential Evolution (DE) [18] as its global search engine. Our main motivation to use DE was that MOEAs based on this search engine have been found to be very effective, outperforming those based on genetic algorithms [21]. Our proposed approach incorporates two additional mechanisms. The first (local dominance) is used to improve the convergence rate towards the Pareto front, while the second (a selection mechanism based on a scalarization function) is used to find nondominated solutions covering the entire Pareto front. To assess the performance of the proposed algorithm, we adopt nine test functions (5 with two objectives and 4 with three objectives), and two performance measures taken from the specialized literature. Our results are compared with respect to the NSGA-II [6] and SPEA2 [25], which are two MOEAs representative of the state-of-the-art in the area.

The remainder of the paper is organized as follows: In Section II some basic multiobjective optimization concepts are introduced. In Section III some previous related work is summarized. Section IV is devoted to describe the proposed approach. Then, the experimental setup is presented in Section V. In Section VI the obtained results are presented and discussed. Finally, in Section VII we provide our conclusions and some possible lines of future work.

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II. Basic Concepts

A Multi-Objective Optimization Problem (MOP) can be mathematically defined as[1]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \dots, m \tag{2}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \dots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \dots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, m$, $j = 1, \dots, p$ are the constraint functions of the problem.

The set of constraints of the problem defines the feasible region in the search space of the problem. Any vector of variables $\vec{x}$ which satisfies all the constraints is considered a feasible solution.

Regarding optimal solutions in MOPs, the following definitions are relevant:

[1] Without loss of generality, minimization is assumed in the following definitions, since any maximization problem can be transformed into a minimization one.

Definition 1. A vector of decision variables $\vec{x} \in \mathbb{R}^n$ dominates another vector of decision variables $\vec{y} \in \mathbb{R}^n$, (denoted by $\vec{x} \prec \vec{y}$) if and only if $\vec{x}$ is partially less than $\vec{y}$, i.e. $\forall i \in \{1,\ldots,k\}, f_i(\vec{x}) \leq f_i(\vec{y}) \wedge \exists i \in \{1,\ldots,k\} : f_i(\vec{x}) < f_i(\vec{y})$.

Definition 2. A vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. A vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

The goal when solving a MOP consists on determining the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3).

III. Previous related work

DE is a simple and powerful evolutionary algorithm that has been found to outperform genetic algorithms in a variety of numerical single-objective optimization problems [18]. DE encodes solutions as vectors and uses operations such as vector addition, scalar multiplication and exchange of components (crossover) to construct new solutions from the existing ones. DE operates as follows: a newly created solution, also called *candidate*, is compared to its parent. If the candidate is better than its parent, it replaces the parent in the population; otherwise, the candidate is discarded. Being a steady-state algorithm, it implicitly enforces *elitism*, i.e., no solution from the population can be deleted unless a better solution is created. DE was originally proposed to deal with real-numbers encoding.

A. Multi-Objective Differential Evolution

DE has been adopted to solve MOPs in several ways. In the earlier approaches (PDE [1] and GDE [14]), only the concept of Pareto dominance was used to compare individuals. The parent was replaced only if it was dominated by the candidate, it was discarded otherwise. Many subsequent approaches (PDEA [15], MODE [22], NSDE [10], GDE2 [12], DEMO [19], GDE3 [13] and NSDE-DCS [11]), use nondominated sorting and/or the crowding distance metric to evaluate the fitness of the individuals. Only recently, new algorithms that do not follow the environmental selection of NSGA-II were proposed ($\epsilon - MyDE$ [20], DEMORS [9], $\epsilon - ODEMO$ [4]). A comprehensive review of some these multi-objective differential evolution approaches can be found in [16].

Algorithm 1 MODE-LD+SS

INPUT:
$P[1,\ldots,N]$ = Population
N = Population Size
F = Scaling factor
CR = Crossover Rate
$\lambda[1,\ldots,N]$ = Weight vectors
NB = Neighborhood Size
$GMAX$ = Maximum number of generations
OUTPUT:
PF = Pareto front approximation
Begin
$g \leftarrow 0$
Randomly create P_i^g , $i = 1,\ldots,N$
Evaluate P_i^g , $i = 1,\ldots,N$
while $g < GMAX$ **do**
  $LND = \{\emptyset\}$
  for $i = 1$ to N **do**
    DetermineLocalDominance(P_i^g,NB_i)
    if P_i^g is locally nondominated **then**
      $\{LND\} \leftarrow \{LND\} \cup P_i^g$
    end if
  end for
  for $i = 1$ to N **do**
    Randomly select $\vec{u_1}$, $\vec{u_2}$, and $\vec{u_3}$ from $\{LND\}$
    $v \leftarrow$ *CreateMutantVector*(u_1, u_2, u_3)
    $P_i^{g+1} \leftarrow$ *Crossover*(P_i^g, v)
    Evaluate P_i^{g+1}
  end for
  $Q \leftarrow P^g \cup P^{g+1}$
  Determine $z*$ for Q
  for $i = 1$ to N **do**
    $P_i^{g+1} \leftarrow$ *MinimumTchebycheff*($Q, \lambda_i, z*$)
    $Q \leftarrow Q \backslash P_i^{g+1}$
  end for
end while
ReturnPF
End

IV. Our Proposed Approach

The MOEA presented in this work (called MODE-LD+SS), adopts the evolutionary operators from differential evolution. In the basic DE algorithm, and during the offspring creation stage, for each current vector $P_i \in \{P\}$, three parents (mutually different among them) $\vec{u_1}, \vec{u_2}, \vec{u_3} \in \{P\}$ ($\vec{u_1} \neq \vec{u_2} \neq \vec{u_3} \neq P_i$) are randomly selected for creating a mutant vector $\vec{v}$ using the following mutation operation:

$$\vec{v} \leftarrow \vec{u_1} + F \cdot (\vec{u_2} - \vec{u_3}) \quad (4)$$

$F > 0$, is a real constant *scaling factor* which controls the amplification of the difference $(\vec{u_2} - \vec{u_3})$. Using this mutant vector, a new offspring $P_i^{'}$ (also called trial vector in DE) is created by crossing over the mutant vector $\vec{v}$ and the current solution P_i, in accordance to:

$$P_j^{'} = \begin{cases} v_j & \text{if } (rand_j(0,1) \leq CR \text{ or } j = j_{rand} \\ P_j & \text{otherwise} \end{cases} \quad (5)$$

In the above expression, the index j refers to the jth component of the decision variables vectors. CR is a positive constant and j_{rand} is a randomly selected integer in the range $[1,\ldots,D]$ (where D is the dimension of the solution vectors) ensuring that the offspring is different at least in one component with respect to the current solution P_i. The above DE variant is known as $Rand/1/bin$, and is the version adopted in the present work. Additionally, the proposed algorithm incorporates two mechanisms for improving both

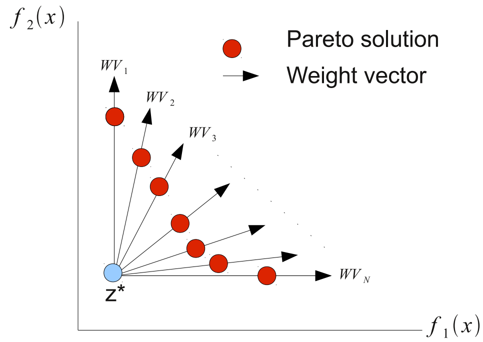

Fig. 1. Distribution of the weight vectors

the convergence towards the Pareto front and the uniform distribution of nondominated solutions along the Pareto front. These mechanisms correspond to the concept of local dominance and the use of an environmental selection based on a scalar function. Below, we explain these two mechanisms in more detail. Algorithm 1 shows the description of our proposed MODE-LD+SS.

In Algorithm 1, the solution vectors $\vec{u_1}, \vec{u_2}, \vec{u_3}$ are selected from the current population, only if they are locally nondominated in their neighborhood $\aleph$. Local dominance is defined as follows:

Definition 6. Pareto Local Dominance Let $\vec{x}$ be a feasible solution, $\aleph(\vec{x})$ be a neighborhood structure for $\vec{x}$, and $\vec{f}(\vec{x})$ a vector of objective functions.

- We say that a solution $\vec{x}$ is locally nondominated with respect to $\aleph(\vec{x})$ if and only if there is no $\vec{x}'$ in the neighborhood of $\vec{x}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$

The neighborhood structure is defined as the NB closest individuals to a particular solution. Closeness is measured by using the Euclidean distance between solutions.

The second mechanism that we introduced is called *selection based on a scalar function*, and is based on the Tchebycheff scalarization function given by:

$$g(\vec{x}|\lambda, z^*) = \max_{1 \le i \le m} \{\lambda_i |f_i(x) - z_i^*|\} \quad (6)$$

In the above equation, $\lambda_i, i = 1, \ldots, N$ represents the weighted vectors used to distribute the solutions along the entire Pareto front (see Figure 1). z^* corresponds to a reference point, defined in objective space and determined with the minimum objective values of the population. This reference point is updated at each generation, as the evolution progresses. The procedure *MinimumTchebycheff($Q, \lambda_i, z*$)* finds, from the set Q (the combined population consistent on the actual parents and the created offspring), the solution vector that minimizes equation (6) for each weight vector λ_i and the reference point z^*.

V. Experimental setup

In order to validate the proposed approach, our results are compared with respect to those of NSGA-II [6] and SPEA2 [25], which are two MOEAs representative of the state-of-the-art in evolutionary multiobjective optimization. Our approach was validated using nine test problems: five from the ZDT (Zitzler-Deb-Thiele) test suite [24] each with 2 objectives (ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6), and four more from the DTLZ (Deb-Thiele-Laumanns-Zitzler) test suite [8], each with 3 objectives (DTLZ1, DTLZ2, DTLZ3, and DTLZ4). The selected test functions comprise different difficulties such as convex, concave, and disconnected Pareto fronts, as well as problems with multiple fronts. The details of these test problems are omitted here due to space constraints, but can be found in [23], [7], [5].

Two performance measures were adopted in order to assess our results: *Hypervolume (Hv)* and *Two Set Coverage (C-Metric)*. A brief description of them is presented next.

A. Hypervolume (Hv):

Given a Pareto approximation set PF_{known}, and a reference point in objective space z_{ref}, this performance measure estimates the *Hypervolume* attained by it. Such hypervolume corresponds to the non-overlaping volume of all the hypercubes formed by the reference point (z_{ref}) and every vector in the Pareto set approximation. This is mathematically defined as:

$$HV = \{\cup_i vol_i | vec_i \in PF_{known}\}$$

vec_i is a nondominated vector from the Pareto set approximation, and vol_i is the volume for the hypercube formed by the reference point and the nondominated vector vec_i. Here, the reference point (z_{ref}) in objective space for the 2-objective MOPs was set to (1.05,1.05), for DTLZ1 was set to (0.6,0.6,0.6), and to (1.05,1.05,1.05) for DTLZ2, DTLZ3 and DTLZ4. This performance measure is Pareto compliant [26], [27], and is used to assess both convergence and distribution of the solutions along the approximated Pareto front. High values indicate that the solutions are closer to the true Pareto front and that they cover a wider extension of it.

B. Two Set Coverage (C-Metric):

This performance measure is also Pareto compliant, and estimates the coverage proportion, in terms of percentage of dominated solutions, between two sets. Given the sets A and B, both containing only nondominated solutions, the C-Metric is mathematically defined as:

$$C(A,B) = \frac{|\{u \in B | \exists v \in A : v \; dominates \; u\}|}{|B|}$$

This metric indicates the portion of vectors in B being dominated by any vector in A. In the present work this measure is used in two different ways. In the first, the set A is the true Pareto front, which is knonw for all test functions; therefore, the C-Metric can be considered as a measure for the ability of the algorithm to find solutions that are nondominated with respect to the Pareto optimal set (i.e., solutions that also belong to the Pareto optimal set). In the second way, sets A and B correspond to two different Pareto approximations, as obtained by two different algorithms.

Therefore, the C-Metric is used for pairwise comparisons between the two algorithms used.

C. Parameters settings:

The parameters used in the experiments for the different algorithms adopted were set as follows. The common parameters for all algorithms comprise the population size N and maximum number of generations $GMAX$. These were set to N = 100 for all the bi-objectives MOPs and N = 300 for all the MOPs having three objectives. We adopted $GMAX$ = 150 for all MOPs, except for ZDT4 and DTLZ3, in which we used $GMAX$ = 200. As for specific parameters of each algorithm, for the NSGA-II, the parameters used were: Crossover probability $p_c = 1.0$; mutation probability $p_m = 1/NVARS$; distribution index for crossover $\eta_c = 15$; distribution index for mutation $\eta_m = 20$. SPEA2 was taken from PISA [2], [3], and was used with the parameters defined therein:

individual_mutation_probability = 1.0;
individual_recombination_probability = 1.0;
variable_mutation_probability = 1/NVARS;
variable_swap_probability = 0.5;
variable_recombination_probability = 0.5;
distribution index for crossover $\eta_c = 15$;
distribution index for mutation $\eta_m = 20$;
use_symmetric_recombination = 0.

For our MODE-LD+SS, the associated parameters were the following: Scaling factor, F = 0.5 for all MOPs; crossover rate, CR = 0.5 for all MOPs, except for ZDT4, where we adopted CR = 0.3; Neighborhood size NB = 5 for all MOPs, except for ZDT4, where NB = 1 was used. The statistics presented for the Hypervolume (Hv) and the C-Metric, when measured with respect to the true Pareto front, were obtained as average values from 32 independent runs for each MOP and for each algorithm. In the case of the statistics for the C-Metric comparing pairs of algorithms (i.e. C-Metric(A,B)), they were obtained as average values of the comparison of all the independent runs from the first algorithm with respect to all the independent runs from the second algorithm.

VI. Results and Discussion

In this section, we present the results obtained by the proposed algorithm MODE-LD+SS, for the nine selected test functions. We also present the comparison with respect to the results attained by NSGA-II and SPEA2.

Table I shows the results obtained for the Hypervolume (Hv) measure for all MOPs, and for the three algorithms compared in this paper. From this table it can be observed that MODE-LD+SS performs better, with respect to the Hv metric, for all the bi-objective MOPs, as compared to NSGA-II and SPEA2. In the case of all the 3-objective MOPs, SPEA2 attains the best results for the Hv measure. However, our proposed MODE-LD+SS obtained values very close to those of SPEA2 in DTLZ1, DTLZ2 and DTLZ4. In those cases, our proposed approach outperforms NSGA-II.

Tables II to X show the comparison matrices for the C-Metric values obtained with the different algorithms and for all the MOPs used in the experiments. The diagonal values of each matrix correspond to the C-Metric for each algorithm, as evaluated with respect to the true Pareto front (i.e. C-Metric(PF_{true},Algorithm)); while the off-diagonal elements correspond to the comparisons between each pair of algorithms. From Tables II to VI, it can be observed that MODE-LD+SS significantly outperforms both NSGA-II and SPEA2 in all the bi-objective problems (ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6). It is also important to note that for ZDT6, our proposed MODE-LD+SS, was able to reach the true Pareto front in the 32 independent runs performed.

For the case of DTLZ1 and DTLZ2, and regarding the C-Metric values presented in Tables VII and VIII, it can be observed that MODE-LD+SS outperforms both NSGA-II and SPEA2. It is important to remark that for these two MOPs, our proposed MODE-LD+SS is able to converge very close from the true Pareto front as indicated by the corresponding convergence measure. These results contrast with the Hv measure obtained by SPEA2 for these same MOPs. The differences can be explained by the fact that SPEA2 obtained a better distribution of solutions. Thus, in this case, one algorithm provided better convergence (MODE-LD+SS), while the other provided better spread of solutions (SPEA2) (see Figures 3). For DTLZ3, SPEA2 attains the best results in terms of the C-Metric (cf. Table IX), while, for this same metric and for DTLZ4, NSGA-II performs better than SPEA2 and MODE-LD+SS (cf. Table X). Finally, Figures 2 and 3 show the comparison of the obtained Pareto fronts by the three MOEAs, for all the MOPs adopted in our study.

VII. Conclusions and Future Work

We have introduced a new MOEA called MODE-LD+SS, which combines differential evolution with local dominance and scalar selection mechanisms. Local dominance aims to improve the convergence rate and the scalar selection mechanism intends to improve the distribution of solutions along the Pareto front. In order to assess the performance of our proposed approach, we adopted 9 test problems and two performance measures (Hypervolume and C-Metric) taken from the specialized literature. Our results were compared with respect to those produced by NSGA-II and SPEA2, which are elitist MOEAs representative of the state-of-the-art in the area.

Our comparative study showed that our proposed MODE-LD+SS outperforms NSGA-II and SPEA2 in 5 of the 9 MOPs used with respect to the Hypervolume, including all the bi-objective MOPs. Our approach was also found to be competitive with respect to SPEA2 in most of the 3-objective MOPs (DTLZ1, DTLZ2 and DTLZ4). Regarding the C-Metric, our proposed MODE-LD+SS outperformed NSGA-II and SPEA2 in 7 of the 9 MOPs adopted. Based on these results, we can conclude that our proposed approach has good convergence properties.

As part of our future work, we are interested in undertaking a thorough statistical analysis of the performance of

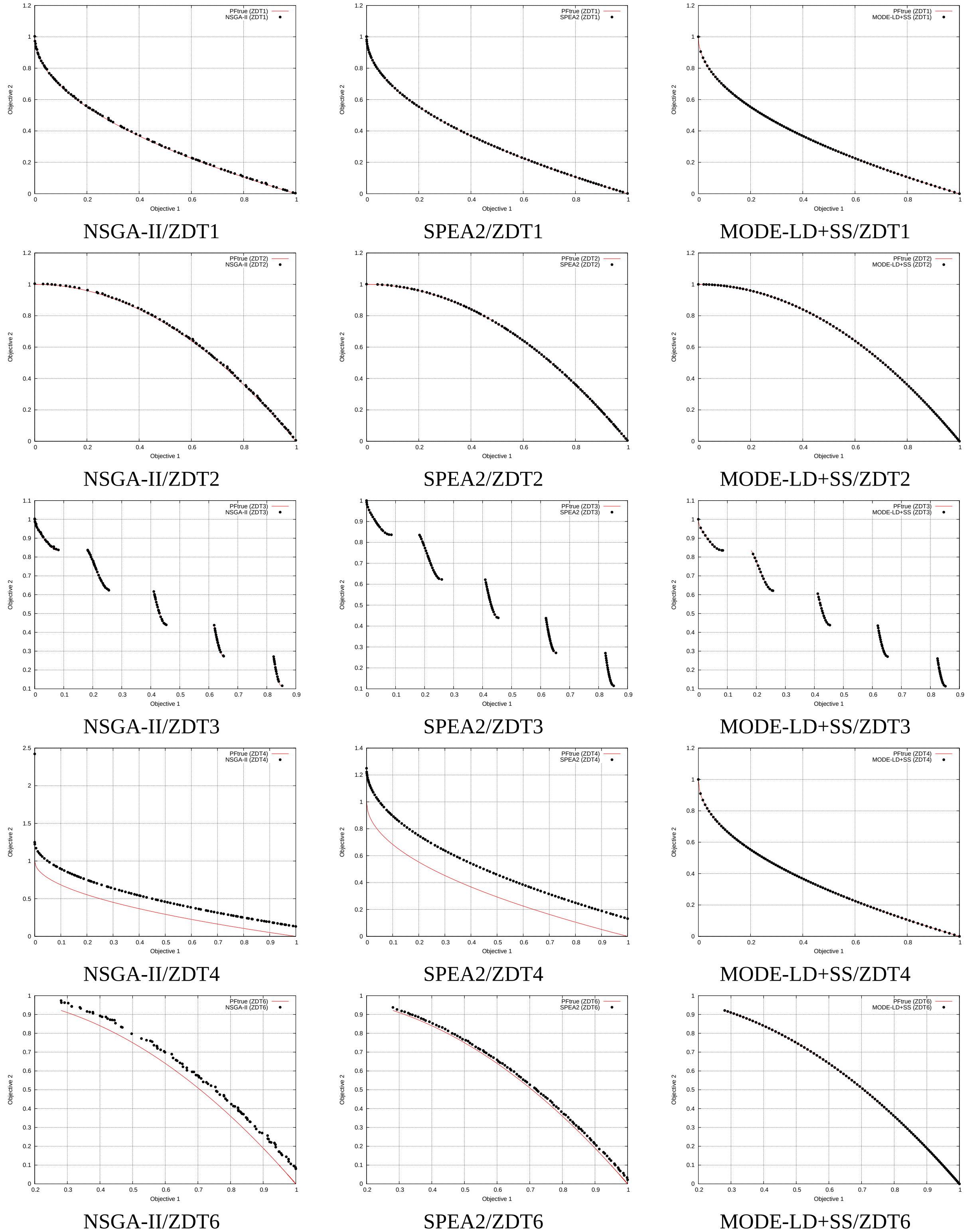

Fig. 2. Pareto fronts obtained by the different algorithms for all the bi-objective MOPs.

TABLE I

COMPARISON OF THE HYPERVOLUME METRIC (HV) FOR ALL THE ALGORITHMS

Test Function	ALGORITHM					
	NSGA		SPEA2		MODE-LD+SS	
	Mean	σ	Mean	σ	Mean	σ
ZDT1	0.757357	0.000928	0.761644	0.000556	**0.763432**	0.000122
ZDT2	0.422221	0.001263	0.321971	0.171286	**0.430341**	0.000144
ZDT3	0.611480	0.008038	0.615533	0.000416	**0.616381**	0.000150
ZDT4	0.217626	0.192914	0.287359	0.188726	**0.741770**	0.058697
ZDT6	0.345949	0.008772	0.392697	0.002336	**0.411054**	0.000003
DTLZ1	0.165918	0.026090	**0.191437**	0.000248	0.187445	0.000347
DTLZ2	0.571146	0.001942	**0.590833**	0.000900	0.581028	0.001193
DTLZ3	0.000000	0.000000	**0.467163**	0.148867	0.000000	0.000000
DTLZ4	0.572327	0.002537	**0.590942**	0.000978	0.577249	0.001997

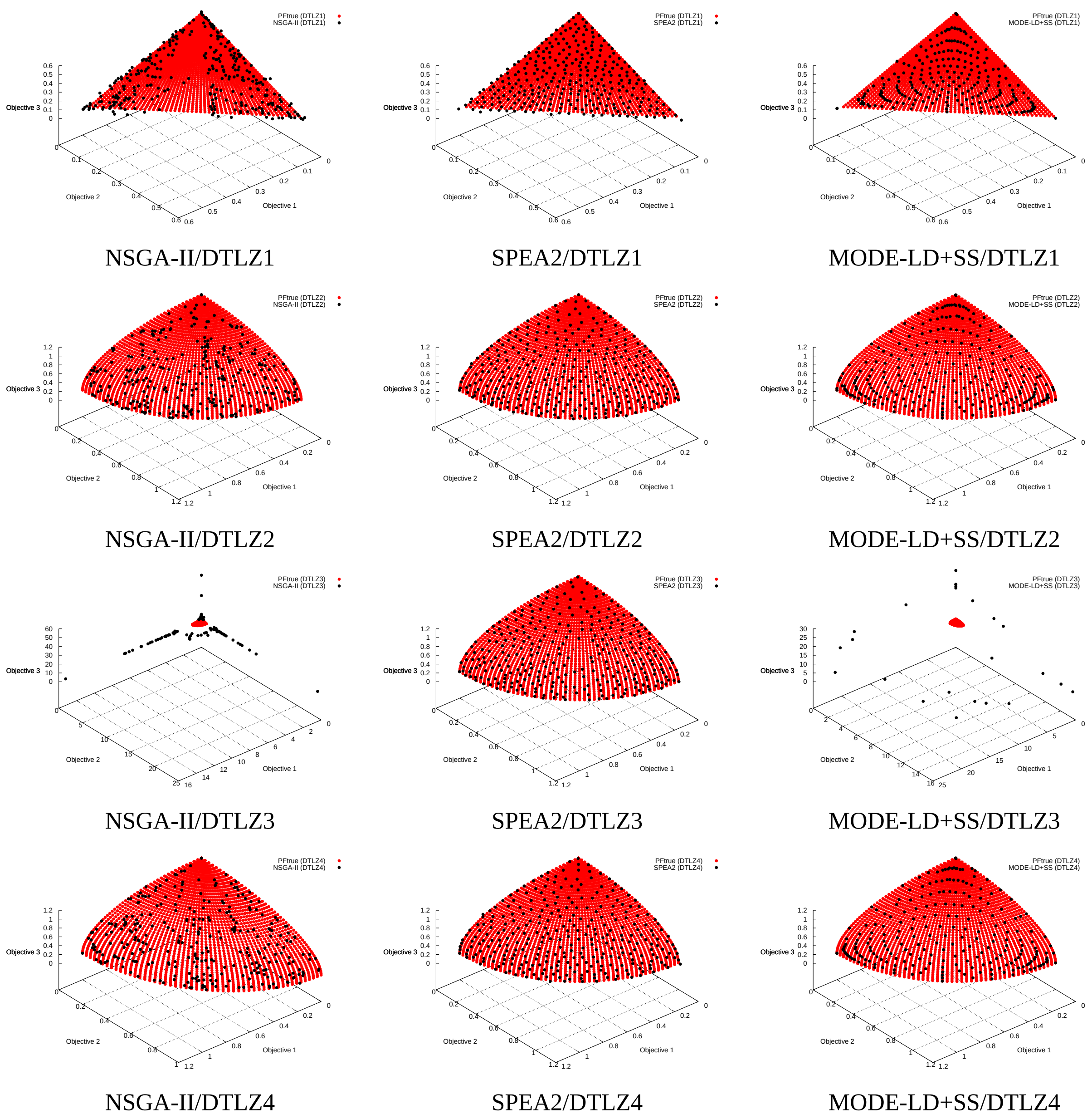

Fig. 3. Pareto fronts obtained by the different algorithms and for all the 3-objective MOPs.

our proposed approach, including an analysis of variance that allows us to determine its most suitable parameter values. We also intend to apply our proposed approach to real-world problems to see if its good convergence properties remain valid in practical applications.

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TABLE II

C-METRIC(A,B) FOR ZDT1

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	0.968750 (0.013854)	0.000771 (0.003989)	0.000000 (0.000000)
SPEA2	0.378115 (0.115819)	0.895000 (0.036100)	0.000000 (0.000000)
MODE-LD+SS	0.589893 (0.088597)	0.214844 (0.064899)	**0.380625** (0.074571)

TABLE III

C-METRIC(A,B) FOR ZDT2

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	1.000000 (0.000000)	0.000303 (0.001877)	0.000000 (0.000000)
SPEA2	0.362813 (0.227343)	0.985938 (0.037232)	0.004331 (0.007530)
MODE-LD+SS	0.702266 (0.086673)	0.242832 (0.167042)	**0.391984** (0.075510)

TABLE IV

C-METRIC(A,B) FOR ZDT3

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	0.656875 (0.075666)	0.002246 (0.008387)	0.000000 (0.000000)
SPEA2	0.339297 (0.106685)	0.389375 (0.065102)	0.000067 (0.000958)
MODE-LD+SS	0.377051 (0.073449)	0.171533 (0.046961)	**0.218731** (0.037570)

TABLE V

C-METRIC(A,B) FOR ZDT4

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	1.000000 (0.000000)	0.301200 (0.455773)	0.000166 (0.001994)
SPEA2	0.546084 (0.489198)	1.000000 (0.000000)	0.000566 (0.003887)
MODE-LD+SS	0.988408 (0.106452)	0.976602 (0.151037)	**0.228750** (0.303674)

TABLE VI

C-METRIC(A,B) FOR ZDT6

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	0.986873 (0.004523)	0.000000 (0.000000)	0.000000 (0.000000)
SPEA2	1.000000 (0.000000)	0.990000 (0.000000)	0.000000 (0.000000)
MODE-LD+SS	0.992119 (0.005944)	0.990000 (0.000000)	**0.000000** (0.000000)

TABLE VII

C-METRIC(A,B) FOR DTLZ1

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean σ)	Mean (σ)	Mean (σ)
NSGA-II	0.655461 (0.143824)	0.001915 (0.003127)	0.000000 (0.000000)
SPEA2	0.707633 (0.234981)	0.258360 (0.100637)	0.000000 (0.000000)
MODE-LD+SS	0.611632 (0.243895)	0.045080 (0.019503)	**0.008116** (0.004630)

TABLE VIII

C-METRIC(A,B) FOR DTLZ2

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	0.354375 (0.031910)	0.027106 (0.009214)	0.000000 (0.000000)
SPEA2	0.044411 (0.012929)	0.806858 (0.029297)	0.000000 (0.000000)
MODE-LD+SS	0.082272 (0.014202)	0.078098 (0.013666)	**0.074566** (0.017111)

TABLE IX

C-METRIC(A,B) FOR DTLZ3

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	1.000000 (0.000000)	0.000221 (0.000868)	0.747556 (0.139952)
SPEA2	0.877437 (0.163622)	**0.798140** (0.107026)	0.916973 (0.131659)
MODE-LD+SS	0.091866 (0.090427)	0.000208 (0.000807)	1.000000 (0.000000)

TABLE X

C-METRIC(A,B) FOR DTLZ4

C-Metric(A,B)	NSGA-II	SPEA2	MODE-LD+SS
	Mean (σ)	Mean (σ)	Mean (σ)
NSGA-II	**0.361563** (0.038679)	0.026370 (0.010174)	0.000000 (0.000000)
SPEA2	0.043145 (0.014076)	0.746696 (0.020173)	0.000000 (0.000000)
MODE-LD+SS	0.077891 (0.019727)	0.077581 (0.016563)	0.516727 (0.021289)